$\boxed{\text{WORKSHEET \#13}}$

$\boxed{\#1}$ $\frac{dy}{dx} = y(y-1)(y-2) = g(y)$

the equilibria are obtained by setting

$g(y)=0$ so $y(y-1)(y-2)=0$

$\Longleftrightarrow$ $\hat{y} = 0, 1, 2$

We need $g'(y)$ to apply the Stability Criterion.

$g(y) = \underbrace{(y^2-y)(y-2)}_{\text{multiply out}} = y^3 - 3y^2 + 2y$

Now $g'(y) = 3y^2 - 6y + 2$

$g'(0) = 2 > 0$ so $\boxed{\hat{y} = 0 \text{ is unstable}}$

$g'(1) = 3(1)^2 - 6(1) + 2 = -1 < 0$ so $\boxed{\hat{y} = 1 \text{ is locally stable}}$

$g'(2) = 3(2)^2 - 6(2) + 2 = 2 > 0$ so $\boxed{\hat{y} = 2 \text{ is unstable}}$

$\boxed{\#2}$ $$\frac{dP}{dt} = P(1-P)\left(\underset{\substack{\uparrow \\ \text{alpha}}}{2}(1-P) - \underset{\substack{\uparrow \\ \text{beta}}}{3}P\right)$$

so $$\boxed{\frac{dP}{dt} = P(1-P)(2-5P) = g(P)}$$

and the equilibria are

$$\hat{P}=0, \quad \hat{P}=1, \quad \hat{P}=\frac{2}{5}=0.4$$

$$g(P) = P(1-P)(2-5P) = (P-P^2)(2-5P)$$
$$= 5P^3 - 7P^2 + 2P$$

$$\boxed{g'(P) = 15P^2 - 14P + 2}$$

Hence by the stability criterion we have

$g'(0) = 2$ so $\hat{P}=0$ is unstable

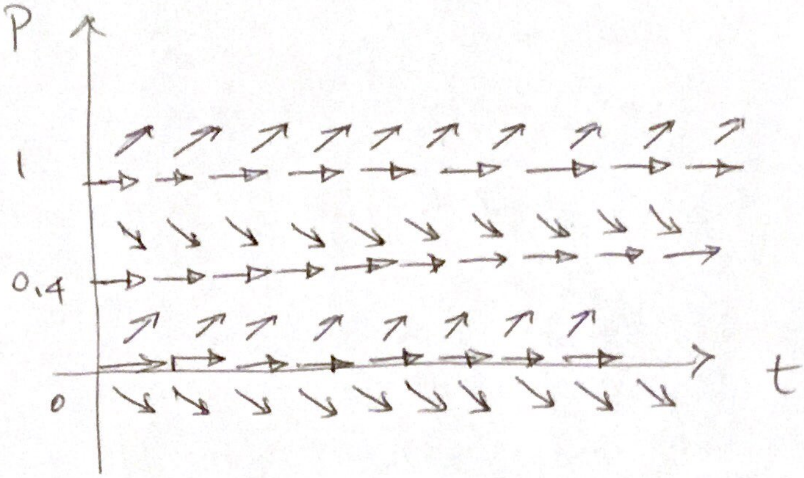
$g'(0.4) = 15(0.4)^2 - 14(0.4) + 2 = -1.2$

so $\hat{P}=0.4$ is locally stable

$g'(1) = 15(1)^2 - 14(1) + 2 = 3$

so $\hat{P}=1$ is unstable

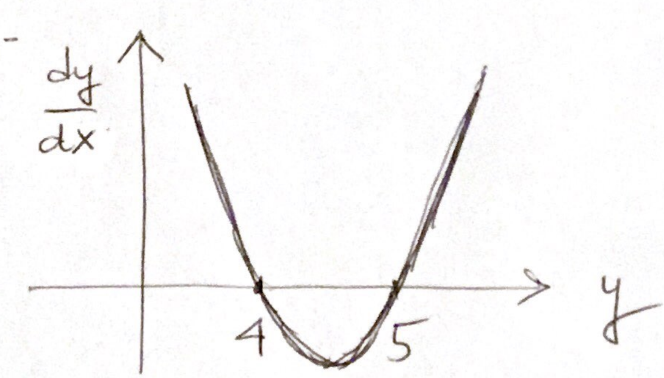
The direction field for $\frac{dP}{dt}$ looks like



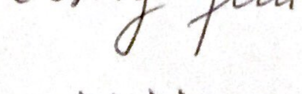
Use SAGE for a better description

$\boxed{\#3}$ $\frac{dy}{dx} = (4-y)(5-y) = g(y)$

The graph of $g(y)$ is a parabola that opens up.

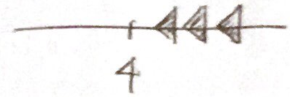


For a $y < 4$ we have $\frac{dy}{dx} > 0$ so $y = y(x)$ is an increasing function. So the motion goes as

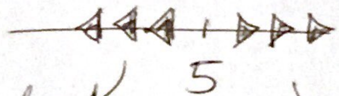


For a $y > 4$ by nearby we have $\frac{dy}{dx} < 0$

so $y = y(x)$ is decreasing so the y values

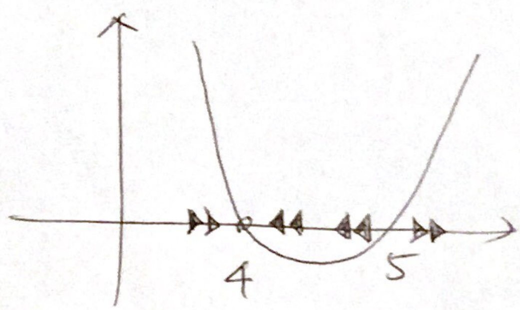
decrease toward 4 

Similarly nearby 5 we have



because $\frac{dy}{dx} < 0$ and y is decreasing

because $\frac{dy}{dx} > 0$ and so y is increasing

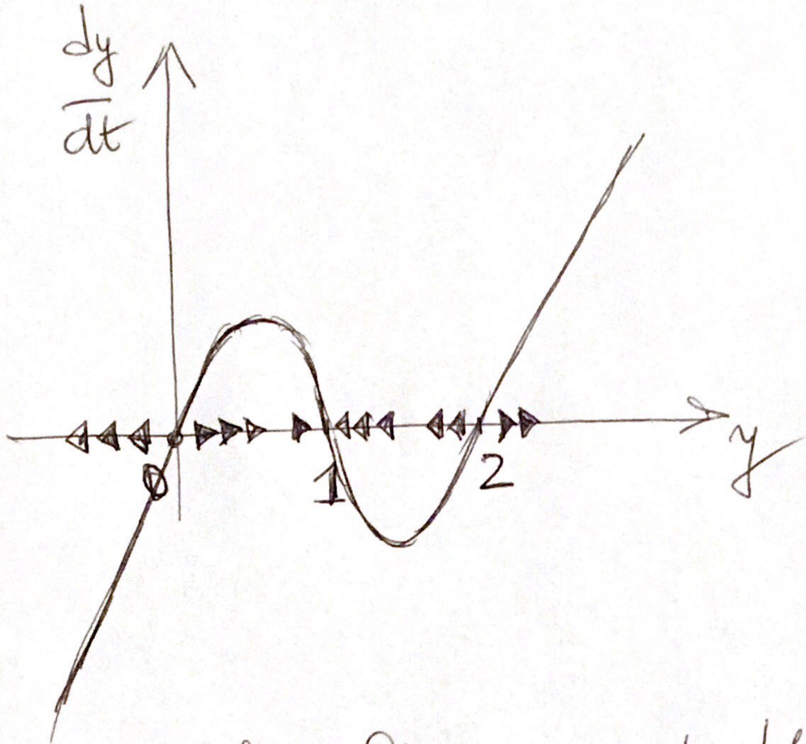


$\hat{y} = 4$ is locally stable

$\hat{y} = 5$ is unstable

$\boxed{\#4}$ $\frac{dy}{dx} = y(y-1)(y-2) = g(y)$

The graph of this cubic function of y is



That's the phase line around those equilibria

$\hat{y} = 0$ is unstable

$\hat{y} = 1$ is locally stable

$\hat{y} = 2$ is unstable