WORKSHEET #15

#1

$$\begin{cases} 9x + 8y = h \\ -4x + ky = -2 \end{cases}$$

Let's consider the augmented matrix

$$\left[\begin{array}{cc|c} 9 & 8 & h \\ -4 & k & -2 \end{array}\right]$$

Perform Gaussian Elimination:

$$\tfrac{1}{9}R_1 \left[\begin{array}{cc|c} 1 & 8/9 & h/9 \\ -4 & k & -2 \end{array}\right]$$

$$R_2 + 4R_1 \left[\begin{array}{cc|c} 1 & 8/9 & h/9 \\ 0 & k + \frac{32}{9} & -2 + 4\frac{h}{9} \end{array}\right]$$

In order to have infinitely many solutions the last row must read $\left[\begin{array}{cc|c} 0 & 0 & 0 \end{array}\right]$

Hence $k + \dfrac{32}{9} = 0$ and $-2 + 4\dfrac{h}{9} = 0$

so $\boxed{k = -32/9}$ and $\boxed{h = 9/2}$

Another solution: write the 2 equations in slope-intercept form.

$$\begin{cases} y = -\frac{9}{8}x + \frac{h}{8} \\ y = \frac{4}{k}x - \frac{2}{k} \end{cases}$$

The two equations must be the same.

$$\underbrace{-\frac{9}{8} = \frac{4}{k}}_{\text{same slope}} \qquad \underbrace{\frac{h}{8} = -\frac{2}{k}}_{\text{same intercept}}$$

$$\underline{\underline{k = -\frac{32}{9}}} \qquad h = -\frac{16}{k} = -\frac{16}{-32/9} = \underline{\underline{\frac{9}{2}}}$$

$\boxed{\#2}$

$$\left[\begin{array}{ccc|c} 3 & 5 & -1 & 10 \\ 2 & -1 & 3 & 9 \\ 4 & 2 & -3 & -1 \end{array}\right]$$

$$\begin{array}{c} \\ \\ R_3 - R_1 \end{array} \left[\begin{array}{ccc|c} 3 & 5 & -1 & 10 \\ 2 & -1 & 3 & 9 \\ 1 & -3 & -2 & -11 \end{array}\right]$$

$$\begin{array}{c}R_3\\ \\ R_1\end{array}\left[\begin{array}{ccc|c}1 & -3 & -2 & -11\\ 2 & -1 & 3 & 9\\ 3 & 5 & -1 & 10\end{array}\right]$$

$$\begin{array}{c} \\ R_2-2R_1\\ R_3-3R_1\end{array}\left[\begin{array}{ccc|c}1 & -3 & -2 & -11\\ 0 & 5 & 7 & 31\\ 0 & 14 & 5 & 43\end{array}\right]$$

$$\begin{array}{c} \\ \tfrac{1}{5}R_2\\ \\ \end{array}\left[\begin{array}{ccc|c}1 & -3 & -2 & -11\\ 0 & 1 & 7/5 & 31/5\\ 0 & 14 & 5 & 43\end{array}\right]$$

$$\begin{array}{c} \\ \\ R_3-14R_2\end{array}\left[\begin{array}{ccc|c}1 & -3 & -2 & -11\\ 0 & 1 & 7/5 & 31/5\\ 0 & 0 & -73/5 & -219/5\end{array}\right]$$

$$\left[\begin{array}{ccc|c}1 & -3 & -2 & -11\\ 0 & 1 & 7/5 & 31/5\\ 0 & 0 & 1 & 3\end{array}\right]$$

$$\begin{array}{c}R_1+2R_3\\ R_2-\tfrac{7}{5}R_3\\ \\ \end{array}\left[\begin{array}{ccc|c}1 & -3 & 0 & -5\\ 0 & 1 & 0 & 2\\ 0 & 0 & 1 & 3\end{array}\right] \rightsquigarrow \begin{array}{c}R_1+3R_2\\ \\ \\ \end{array}\left[\begin{array}{ccc|c}1 & 0 & 0 & 1\\ 0 & 1 & 0 & 2\\ 0 & 0 & 1 & 3\end{array}\right]$$

So $x=1$

$y=2$

$z=3$

#3

$$\left[\begin{array}{ccc|c} 1 & 1 & 3 & -3 \\ 1 & 2 & -3 & -1 \\ 5 & 12 & k & 0 \end{array}\right]$$

$$\begin{array}{l} \\ R_2 - R_1 \\ R_3 - 5R_1 \end{array} \left[\begin{array}{ccc|c} 1 & 1 & 3 & -3 \\ 0 & 1 & -6 & 2 \\ 0 & 7 & k-15 & 15 \end{array}\right]$$

$$\begin{array}{l} \\ \\ R_3 - 7R_2 \end{array} \left[\begin{array}{ccc|c} 1 & 1 & 3 & -3 \\ 0 & 1 & -6 & 2 \\ 0 & 0 & k+27 & 1 \end{array}\right]$$

For the system not to have solutions we must have that the last equation reads

$$\underline{\underline{0 \cdot z = 1}}$$

impossible equat.

so $k + 27 = 0$

or $\boxed{k = -27}$

$\boxed{\#4}$ the augmented matrix is

$$\left[\begin{array}{cc|c} 2 & -3 & 5 \\ 4 & -6 & c \end{array}\right]$$

Let's use the Gaussian Elimination

$$\begin{array}{l} \tfrac{1}{2}R_1 \\ \\ \end{array} \left[\begin{array}{cc|c} 1 & -3/2 & 5/2 \\ 4 & -6 & c \end{array}\right]$$

$$\begin{array}{l} \\ R_2-4R_1 \end{array} \left[\begin{array}{cc|c} 1 & -3/2 & 5/2 \\ 0 & 0 & c-10 \end{array}\right]$$

The system will have infinitely many solutions when $c-10=0$

OR $c=10$. This means that there is only one equation.

For $c \neq 10$ there is no solution.

There is no way we can only have one solution as the two lines are parallel