WORKSHEET #16

#1

$$\left[\begin{array}{ccc|c} 1 & 4 & 3 & 8 \\ 1 & 2 & -1 & 2 \\ 3 & 8 & 1 & 12 \end{array}\right]$$

$$\begin{array}{c} \\ R_2 - R_1 \\ R_3 - 3R_1 \end{array} \left[\begin{array}{ccc|c} 1 & 4 & 3 & 8 \\ 0 & -2 & -4 & -6 \\ 0 & -4 & -8 & -12 \end{array}\right]$$

$$\begin{array}{c} \\ -\frac{1}{2}R_2 \\ \\ \end{array} \left[\begin{array}{ccc|c} 1 & 4 & 3 & 8 \\ 0 & 1 & 2 & 3 \\ 0 & -4 & -8 & -12 \end{array}\right]$$

$$\begin{array}{c} \\ \\ R_3 + 4R_2 \end{array} \left[\begin{array}{ccc|c} 1 & 4 & 3 & 8 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -5 & -4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Hence the system has infinitely many solutions

$$\begin{cases} x \qquad -5z = -4 \\ \quad y + 2z = 3 \end{cases}$$

Hence if we give to z any value t we get $\boxed{x = -4 + 5t, \quad y = 3 - 2t \text{ and } z = t}$. These form a line of solutions.

#2

$$\left[\begin{array}{cc|c} -9 & 6 & 0 \\ -18 & k & -3 \end{array}\right]$$

$$-\tfrac{1}{9}R_1 \left[\begin{array}{cc|c} 1 & -2/3 & 0 \\ -18 & k & -3 \end{array}\right]$$

$$R_2 + 18R_1 \left[\begin{array}{cc|c} 1 & -2/3 & 0 \\ 0 & k-12 & -3 \end{array}\right]$$

~~k must be different from 12 to be consistent~~

#3

$$3\begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} - \frac{1}{2}\begin{bmatrix} 4 & -2 \\ 0 & 8 \\ -6 & 14 \end{bmatrix}$$

*

$$= \begin{bmatrix} 3 & 6 \\ 0 & 3 \\ 3 & 0 \end{bmatrix} + \begin{bmatrix} -2 & 1 \\ 0 & -4 \\ 3 & -7 \end{bmatrix}$$

$$= \begin{bmatrix} 3-2 & 6+1 \\ 0+0 & 3-4 \\ 3+3 & 0-7 \end{bmatrix} = \begin{bmatrix} 1 & 7 \\ 0 & -1 \\ 6 & -7 \end{bmatrix}$$

* $A \cdot C = \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 7 & -1 & 0 \\ 2 & 0 & 1 \\ 3 & -1 & -2 \end{bmatrix}$$

* $C \cdot A = \begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$

$$= \begin{bmatrix} 1 & 5 \\ 3 & 4 \end{bmatrix}$$

$\boxed{\#4}$

$*$ $$\begin{bmatrix} -5 & 2 & 7 \\ 1 & 2 & 0 \\ 0 & 9 & -4 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -5-2 & 2 & 7 \\ 1 & 2-4 & 0 \\ 0 & 9 & -4-6 \end{bmatrix} = \begin{bmatrix} -7 & 2 & 7 \\ 1 & -2 & 0 \\ 0 & 9 & -10 \end{bmatrix}$$

$*$ $$A^2 = \begin{bmatrix} -5 & 2 & 7 \\ 1 & 2 & 0 \\ 0 & 9 & -4 \end{bmatrix} \cdot \begin{bmatrix} -5 & 2 & 7 \\ 1 & 2 & 0 \\ 0 & 9 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} 25+2 & -10+4+63 & -35-28 \\ -5+2 & 2+4 & 7 \\ 9 & 18-36 & 16 \end{bmatrix}$$

$$= \begin{bmatrix} 27 & 57 & -63 \\ -3 & 6 & 7 \\ 9 & -18 & 16 \end{bmatrix}$$

$*$ $$B^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

* $B^4 = B^2 \cdot B^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 9 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 9 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 16 & 0 \\ 0 & 0 & 81 \end{bmatrix}$

* $AB = \begin{bmatrix} -5 & 2 & 7 \\ 1 & 2 & 0 \\ 0 & 9 & -4 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

$= \begin{bmatrix} -5 & 4 & 21 \\ 1 & 4 & 0 \\ 0 & 18 & -12 \end{bmatrix}$

* $A^T = \begin{bmatrix} -5 & 1 & 0 \\ 2 & 2 & 9 \\ 7 & 0 & -4 \end{bmatrix}$

$\boxed{\#5}$

$$\begin{bmatrix} 2 & 4a \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 0 & 6-a \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} -40 & 2 \\ 8 & b \end{bmatrix}$$

$$\begin{bmatrix} 8a & 12+2a \\ 8 & 16-2a \end{bmatrix} = \begin{bmatrix} -40 & 2 \\ 8 & b \end{bmatrix}$$

$\Rightarrow 8a = -40$ OR $\boxed{a = -5}$ $\boxed{16 + 10 = \overset{26}{\overset{\shortparallel}{b}}}$