WORKSHEET #18

1. $\underline{u} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ $\quad \underline{v} = \begin{bmatrix} -1 \\ -3 \end{bmatrix}$ $\quad \underline{w} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$

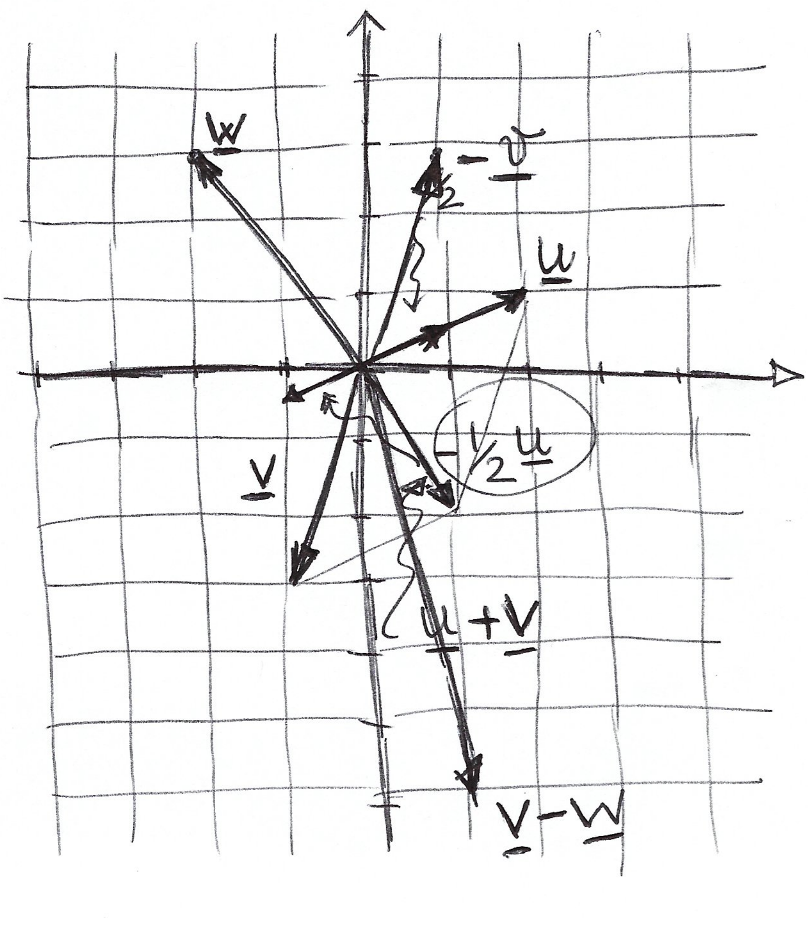
$$-\tfrac{1}{2}\underline{u} = -\tfrac{1}{2}\begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -\tfrac{1}{2}\cdot 2 \\ -\tfrac{1}{2}\cdot 1 \end{bmatrix} = \begin{bmatrix} -1 \\ -\tfrac{1}{2} \end{bmatrix}$$

$$\underline{u} + \underline{v} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} + \begin{bmatrix} -1 \\ -3 \end{bmatrix} = \begin{bmatrix} 2-1 \\ 1-3 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$$\underline{v} - \underline{w} = \begin{bmatrix} -1 \\ -3 \end{bmatrix} - \begin{bmatrix} -2 \\ 3 \end{bmatrix} = \begin{bmatrix} -1+2 \\ -3-3 \end{bmatrix} = \begin{bmatrix} 1 \\ -6 \end{bmatrix}$$

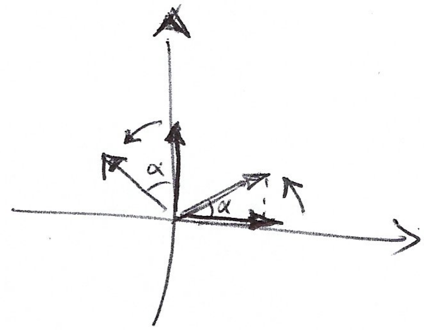
$$-\underline{v} = -\begin{bmatrix} -1 \\ -3 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

Geometrically,



#2 A general rotation matrix is of the form

$$R_\alpha = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$



A counterclockwise rotation corresponds to a positive α

A clockwise rotation corresponds to a negative α

In our case $\alpha = \pi/2$, so

$$R_{\pi/3} = \begin{bmatrix} \cos \pi/3 & -\sin \pi/3 \\ \sin \pi/3 & \cos \pi/3 \end{bmatrix} = \begin{bmatrix} 1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{bmatrix}$$

$$\begin{bmatrix} 1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 1/2 - 3/2\sqrt{3} \\ \sqrt{3}/2 + 3/2 \end{bmatrix}$$

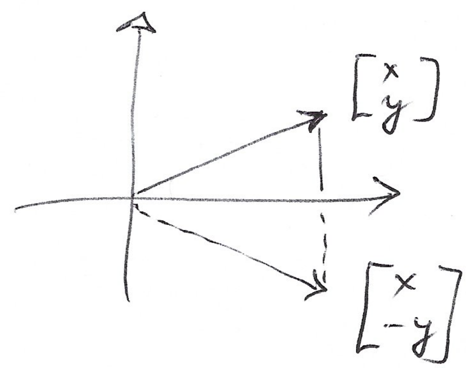
$$\approx \begin{bmatrix} -2.098 \\ 2.366 \end{bmatrix}$$

(c) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is the identity transformation A

as $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\underline{\underline{\begin{bmatrix} x \\ y \end{bmatrix}}} = \underline{\underline{\begin{bmatrix} x \\ y \end{bmatrix}}}$

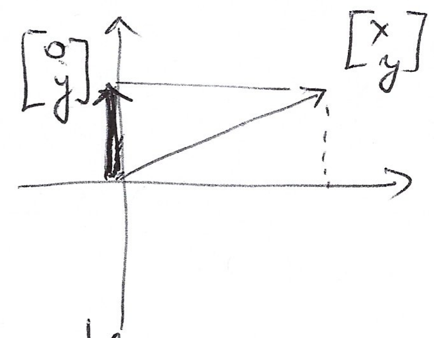
(a) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ is the reflection in the x-axis B

as $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ -y \end{bmatrix}$



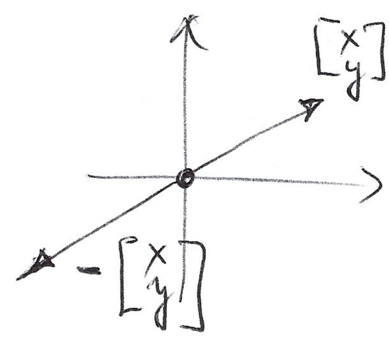
(b) $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ is the projection onto the y-axis D

as $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ y \end{bmatrix}$



(d) $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ is the reflection in the origin C

as $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -x \\ -y \end{bmatrix} = -\begin{bmatrix} x \\ y \end{bmatrix}$



(e) $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} \cos \pi/2 & -\sin \pi/2 \\ \sin \pi/2 & \cos \pi/2 \end{bmatrix}$

it is a counterclockwise rotation of $90°$ degrees (F)

(f) $\begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$ is a contraction by a factor of 2 (E)

as $\begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}\underline{\begin{bmatrix} x \\ y \end{bmatrix}} = \begin{bmatrix} 0.5x \\ 0.5y \end{bmatrix} = \underline{\frac{1}{2}\begin{bmatrix} x \\ y \end{bmatrix}}$

#4

$$\det \begin{bmatrix} 1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{bmatrix} = \frac{1}{2}\cdot\frac{1}{2} - \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$$

$$= \frac{1}{4} + \frac{3}{4} = \frac{4}{4} = \underline{\underline{1}}$$

in general

$$\det \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} = \cos^2\alpha - (-\sin^2\alpha)$$

$$= \cos^2\alpha + \sin^2\alpha$$

$$= \underline{\underline{1}}$$

$\boxed{\#5}$

* $\begin{bmatrix} 68 & 30 \\ -150 & -67 \end{bmatrix} \begin{bmatrix} 2 \\ 7 \end{bmatrix} = \begin{bmatrix} 68 \cdot 2 + 7 \cdot 30 \\ -150 \cdot 2 - 67 \cdot 7 \end{bmatrix}$

$= \begin{bmatrix} 346 \\ -769 \end{bmatrix} \neq \lambda \begin{bmatrix} 2 \\ 7 \end{bmatrix}$

there is no value λ for which this is true.

---

* $\begin{bmatrix} 22 & -9 \\ 30 & -11 \end{bmatrix} \begin{bmatrix} -1 \\ -2 \end{bmatrix} = \begin{bmatrix} -22 + 18 \\ -30 + 22 \end{bmatrix} = \begin{bmatrix} -4 \\ -8 \end{bmatrix}$

$= 4 \begin{bmatrix} -1 \\ -2 \end{bmatrix}$

$\begin{bmatrix} -1 \\ -2 \end{bmatrix}$ eigenvector for A associated with the eigenvalue $\lambda = 4$

* $\begin{bmatrix} 4 & -8 \\ 0 & -4 \end{bmatrix} \begin{bmatrix} -1 \\ -1 \end{bmatrix} = \begin{bmatrix} -4 + 8 \\ +4 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \end{bmatrix} =$

$= -4 \begin{bmatrix} -1 \\ -1 \end{bmatrix}$

$\begin{bmatrix} -1 \\ -1 \end{bmatrix}$ eigenvalue for A associated with the eigenvalue $\lambda = -4$