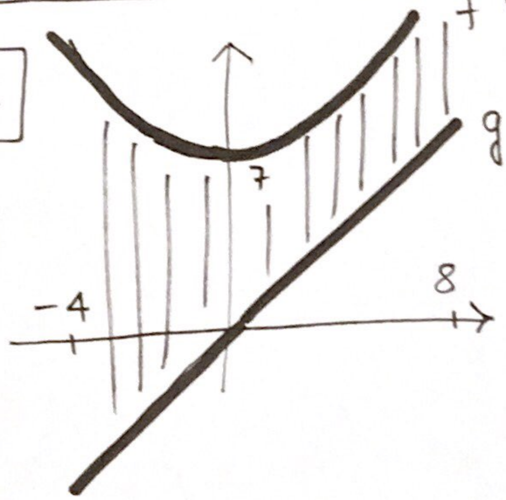
WORKSHEET #1

#2



First of all, let's check that the graphs of $f(x)$ and $g(x)$ do not intersect

$$\begin{cases} y = 0.9x^2 + 7 \\ y = x \end{cases} \Longleftrightarrow \quad x = 0.9x^2 + 7$$

$$\Longleftrightarrow \quad 0.9x^2 - x + 7 = 0$$

using the quadratic formula:

$$x_{1,2} = \frac{1 \pm \sqrt{1 - 4 \cdot 7 \cdot 0.9}}{2 \cdot 0.9}$$ has complex solutions!

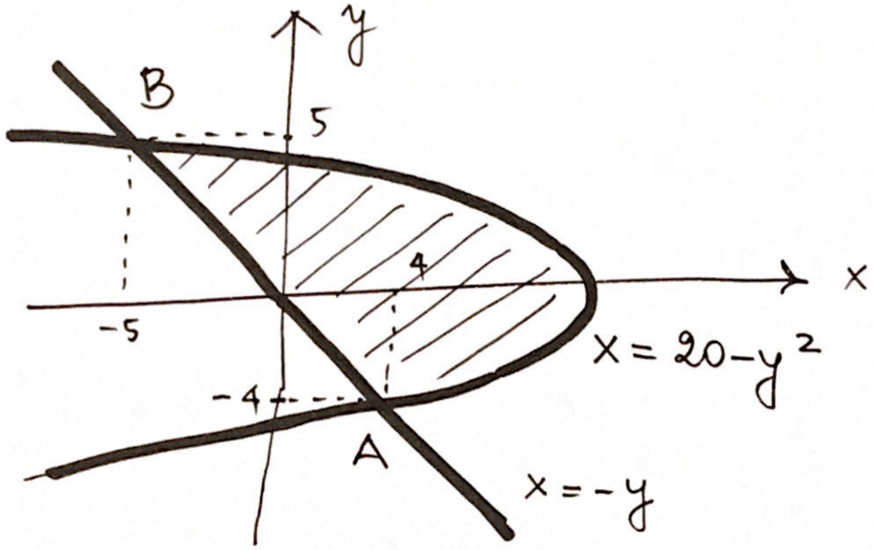
$$\text{Area} = \int_{-4}^{8} \left[(0.9x^2 + 7) - x \right] dx =$$

$$= 0.9 \frac{1}{3} x^3 - \frac{1}{2} x^2 + 7x \Big|_{-4}^{8} =$$

$$= \left[0.3 (8)^3 - \frac{1}{2} (8)^2 + 7 \cdot 8 \right] - \left[0.3 (-4)^3 - \frac{1}{2} (-4)^2 + 7(-4) \right]$$

$$= \boxed{232.8}$$

$\boxed{\#3}$ $x+y=0$ & $x+y^2=20$



Those are the graphs of the two curves.
These are functions of "y".

Let's find their points of intersection.

$$\begin{cases} x=-y \\ x=20-y^2 \end{cases} \iff -y = 20-y^2$$

$$\iff y^2-y-20=0 \iff (y-5)(y+4)=0$$

so $\boxed{y=-4}$ & $\boxed{y=5}$

The corresponding points are $A(4,-4)$ and $B(-5,5)$ as they lie on the line $x=-y$.

It is much more convenient to integrate

with respect to y.

$$\text{Area} = \int_{-4}^{5} \left[(20-y^2) - (-y)\right] dy$$

$$= \int_{-4}^{5} (20 + y - y^2)\, dy$$

$$= 20y + \frac{1}{2}y^2 - \frac{1}{3}y^3 \Big|_{-4}^{5}$$

$$= \left[\left(20.5 + \frac{1}{2}5^2 - \frac{1}{3}5^3\right) - \left(20(-4) + \frac{1}{2}(-4)^2 - \frac{1}{3}(-4)^3\right)\right]$$

$$= \boxed{121.5}$$

$\boxed{\#4}$ The average value of $f(x) = -\frac{8}{x}$ on $[1,4]$ is

$$f_{avg} = \frac{1}{4-1} \cdot \int_{1}^{4} \left(-\frac{8}{x}\right) dx = -\frac{8}{3} \int_{1}^{4} \frac{1}{x}\, dx$$

$$= -\frac{8}{3}\left[\ln|x| \Big|_{1}^{4}\right] = -\frac{8}{3}\left(\ln|4| - \cancel{\ln|1|}_{0}\right)$$

$$= \boxed{-\frac{8}{3}\ln 4} = \underline{-3.696785}$$

$\boxed{\#5}$ $T(t) = 50 + 6\sin\left(\frac{\pi}{12}t\right)$

t = hours after 9 am

(a) temperature at 9 pm : $T(0) = \underline{50}$

since $\sin(0) = 0$

(b) temperature at 3 pm ; it means $t = 6$

$$T(6) = 50 + 6 \underbrace{\sin\left(\frac{\pi}{12}\cdot 6\right)}_{\sin(\pi/2)=1} = \underline{56}$$

(c) Average temperature on $[\underset{9\,am}{0}, \underset{9\,pm}{12}]$

$$T_{avg} = \frac{1}{12}\int_0^{12}\left[50 + 6\sin\left(\frac{\pi}{12}t\right)\right]dt$$

$$= \frac{1}{12}\left[50t + 6\,\frac{1}{\pi/12}\cdot\left(-\cos\left(\frac{\pi}{12}t\right)\right)\Big|_0^{12}\right]$$

$$= \frac{1}{12}\left[\left(50\cdot 12 - \frac{72}{\pi}\cos(\pi)\right) - \left(0 - \frac{72}{\pi}\cos(0)\right)\right]$$

$$= \frac{1}{12}\left[50\cdot 12 + \frac{72}{\pi} + \frac{72}{\pi}\right] = \boxed{50 + \frac{12}{\pi}}$$

$\approx \underline{53.8197\ °F}$

$\boxed{\#6}$

(a) $$v(t) - v(0) = \int_0^t a(u)\,du$$
$$= \int_0^t \frac{dv}{du} \cdot du$$
$$= \int_0^t 32\,du$$
$$= 32t$$

Hence $\boxed{v(t) = v(0) + 32t}$

(b) Since $v(0) = 5$ ft/s $\quad \underline{v(t) = 5 + 32t}$

(c) $\frac{dp}{dt} = v(t)$ where $p(t) = $ position

$$p(t) - p(0) = \int_0^t v(u)\,du$$
$$= \int_0^t (5 + 32u)\,du =$$
$$= 5u + 16u^2 \Big|_0^t = \underline{5t + 16t^2}$$

when $t = 10$ $\quad p(10) - p(0) = 5 \cdot 10 + 16 \cdot 10^2 = \boxed{1{,}650}$