WORKSHEET #24

#1 $f(x,y) = xye^{-5y}$

$f_x = 1 \cdot ye^{-5y} = \underline{ye^{-5y}}$

$f_{xy} = 1 \cdot e^{-5y} + y \cdot e^{-5y}(-5) =$

$= \underline{e^{-5y}(1-5y)}$

$f_y = x \cdot 1 \cdot e^{-5y} + xy \cdot e^{-5y}(-5) =$

$= \underline{xe^{-5y}(1-5y)}$

$f_{yx} = 1 \cdot e^{-5y}(1-5y) = \underline{e^{-5y}(1-5y)}$

$=$

#2 $f(x,y) = \ln(x^3 + y^5)$

$\frac{\partial f}{\partial x} = \frac{1}{x^3+y^5} \cdot 3x^2$; $\frac{\partial f}{\partial y} = \frac{1}{x^3+y^5} \cdot (5y^4)$

$\boxed{\#3}$ Approximate $\left.\dfrac{\partial f}{\partial T}\right|_{(T=96,\,H=70)}$

$$= \lim_{h \to 0} \frac{f(T_0+h, H_0) - f(T_0, H_0)}{h}$$

$$= \lim_{h \to 0} \frac{f(96+h, 70) - f(96, 70)}{h}$$

$$\approx \frac{f(96+h, 70) - f(96, 70)}{h}$$

for small values of h

Since $H=70$ is fixed we need to look at the values in the vertical column corresponding to $H=70$.

$h=2$ $\qquad \dfrac{f(98,70) - f(96,70)}{2} = \dfrac{133-125}{2}$

$$\approx 4$$

$h = -2$ $$\frac{f(94,70) - f(96,70)}{-2} = \frac{118 - 125}{-2}$$

$$\approx \frac{-7}{-2} = 3.5$$

We could take the average of 3.5 and 4. Hence we can approximate

$$\left.\frac{\partial f}{\partial T}\right|_{(96,70)} \approx \frac{3.5+4}{2} = \underline{\underline{3.75}}$$

$\boxed{\#4}$ $$P_e = \frac{aNT}{1 + aT_h N}$$

$$\frac{\partial P_e}{\partial T} = \frac{aN}{1 + aT_h N}$$

$$\frac{\partial P_e}{\partial N} = \frac{aT(1 + aT_h N) - aNT(aT_h)}{(1 + aT_h N)^2}$$

$$= \frac{aT + a^2 \cancel{T T_h N} - a^2 \cancel{N T T_h}}{(1 + aT_h N)^2}$$

$$= \frac{aT}{(1+aT_h N)^2}$$

$$\frac{\partial P_e}{\partial T_h} = \frac{0\cdot(1+aT_h N) - aNT(aN)}{(1+aT_h N)^2}$$

$$= \frac{-a^2N^2T}{(1+aT_h N)^2}$$

#5 $z = f(x,y) = 9x^5 + 7y^5 + 2xy$

at $x_0 = 1$, $y_0 = -3$ $\qquad z_0 = f(1,-3) = -1{,}698$

$$\frac{\partial f}{\partial x} = 45x^4 + 2y \qquad \left.\frac{\partial f}{\partial x}\right|_{(1,-3)} = 45 - 6 = \underline{39}$$

$$\frac{\partial f}{\partial y} = 35y^4 + 2x \qquad \left.\frac{\partial f}{\partial y}\right|_{(1,-3)} = 35(-3)^4 + 2 = 2837$$

Tg plane: $\boxed{z - (-1698) = 39(x-1) + 2837(y+3)}$