WORKSHEET #2

#1

$\frac{dN}{dt}$ = rate of growth of a population at time t

$$\int_{13}^{42} \left(\frac{dN}{dt}\right) \cdot dt$$ = represents the change of the population over the interval $[13, 42]$

$$= N(42) - N(13)$$

#2

$$\frac{dL}{dt} = L_o e^{-kt}$$

k, L_o positive constants

rate of growth of the length of a fish

$$\boxed{L(3) - L(0)} = \int_0^3 \frac{dL}{dt} \cdot dt = \int_0^3 L_o e^{-kt} dt$$

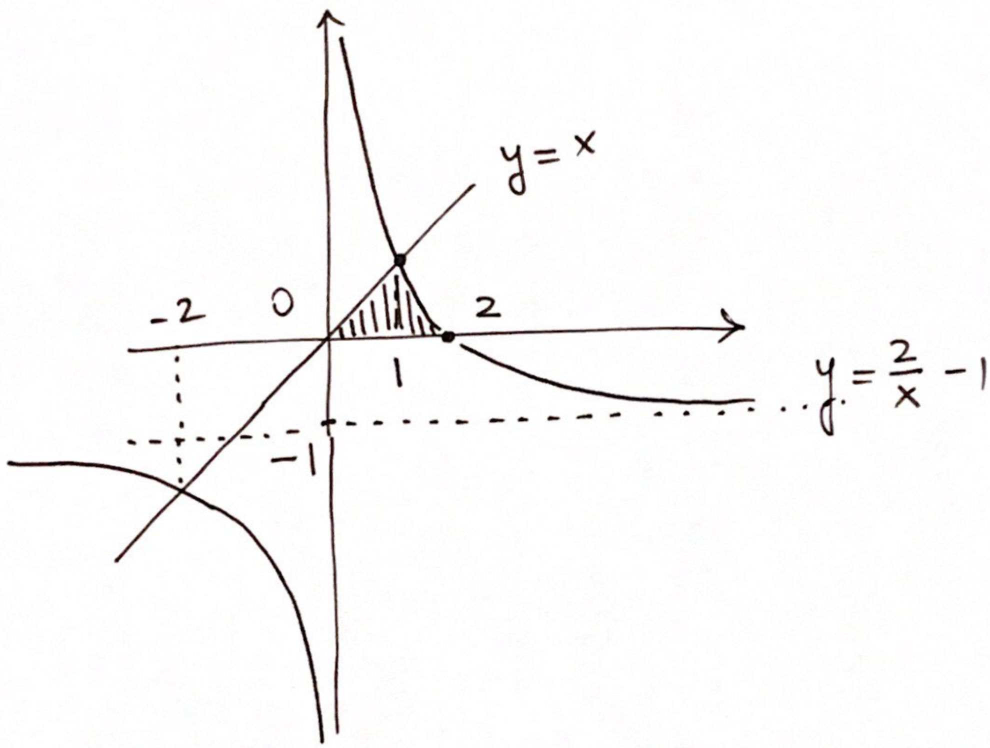
$$= L_o \cdot \left[-\frac{1}{k} e^{-kt} \right] \Big|_0^3 =$$

$$= L_o \cdot \left(-\frac{e^{-3k}}{k} - \left(-\frac{1}{k} \underbrace{e^{0}}_{1} \right) \right) =$$

$$= \boxed{L_o \left(\frac{1}{k} - \frac{e^{-3k}}{k} \right)}$$

$\boxed{\#3}$

$y = x$ & $y = \frac{2}{x} - 1$ & x-axis



Note that $y = \frac{2}{x} - 1$ intersects the x-axis when $y=0$; so $0 = \frac{2}{x} - 1 \iff \frac{2}{x} = 1$

$\iff x = 2.$

Also the intersection point of the 2 curves

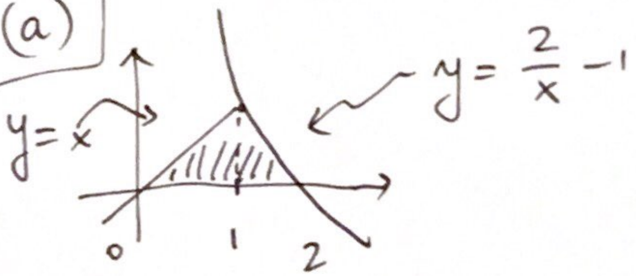
$$\begin{cases} y = x \\ y = \frac{2}{x} - 1 \end{cases} \iff x = \frac{2}{x} - 1$$

$$\iff x + 1 = \frac{2}{x} \iff x^2 + x = 2$$

$$\iff x^2 + x - 2 = 0 \iff (x+2)(x-1) = 0$$

$$\therefore x = 1, -2$$

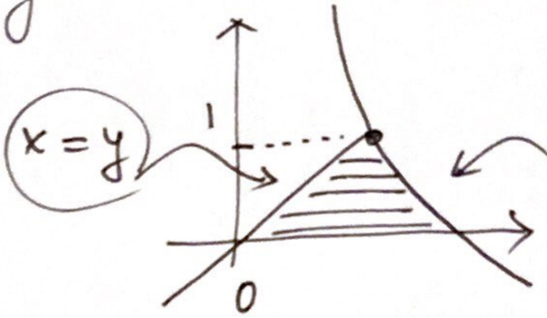
$\boxed{(a)}$



Hence area (as a function of x)

$$\boxed{= \int_0^1 x \cdot dx + \int_1^2 \left(\frac{2}{x} - 1\right) dx}$$

$\boxed{(b)}$ If we view those 2 graphs as functions of y we obtain:



$$y = \frac{2}{x} - 1$$

$$\iff y + 1 = \frac{2}{x}$$

$$\iff \boxed{x = \frac{2}{y+1}}$$

$$\boxed{\text{Area} = \int_0^1 \left(\frac{2}{y+1} - y\right) dy}$$

(c) the second integral is easier to compute

$$= 2 \cdot \ln|y+1| - \frac{1}{2} y^2 \Big|_0^1 =$$

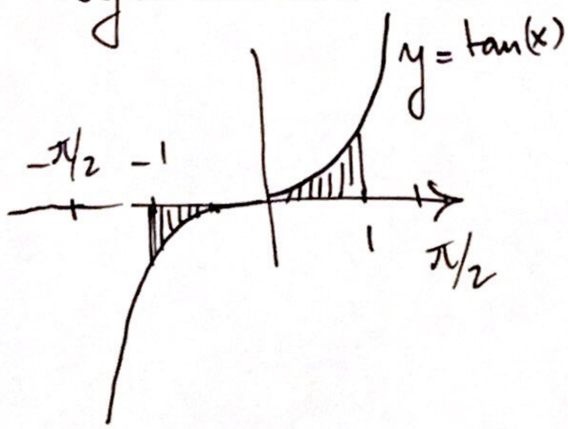
$$= \left[2 \cdot \ln(2) - \tfrac{1}{2}(1)^2\right] - \left[2 \underbrace{\ln(1)}_{0} - \underbrace{\tfrac{1}{2}(0)^2}_{0}\right]$$

$$= 2 \cdot \ln(2) - \frac{1}{2} = \ln(2^2) - \tfrac{1}{2} \approx 0.88629$$

properties of logarithms

#4 Read the explanation on pages 3 through 5 of Lectures 1 & 2.

#5 Look at the graph of $y = \tan x$ over the symmetric interval $[-1, 1]$



the function $\tan(x)$ is <u>odd</u>

i.e. it is symmetric wrt. the origin

i.e. $\tan(-x) = -\tan x$

The area on $[-1, 0]$ and on $[0, 1]$ are opposite in sign.

So $\displaystyle\int_{-1}^{1} \tan x \, dx = 0$

Thus average of $\tan x$ on $[-1, 1] =$

$$= \frac{1}{2} \int_{-1}^{1} \tan x \, dx = \underline{\underline{0}}$$

<u>Check</u> algebraically (it is a <u>tricky integral</u>)

$$\frac{1}{2} \int_{-1}^{1} \tan x \, dx = \int_{-1}^{1} \frac{1}{\cos x} \cdot \sin(x) \, dx$$

$$= \frac{-1}{2} \int_{-1}^{1} \underbrace{\frac{1}{\cos(x)} \left(-\sin(x)\right)}_{\text{this function is the derivative of } \ln|\cos(x)|} dx$$

$$= -\frac{1}{2} \left[\ln|\cos(x)| \Big|_{-1}^{1} \right] =$$

$$= -\frac{1}{2} \left[\ln|\cos(1)| - \ln|\cos(-1)| \right] = 0$$

as $\cos(1) = \cos(-1)$ as <u>$\cos(x)$</u> is an <u>even function</u>