WORKSHEET #4

#1

$$\int \underbrace{2p^5}_{g'} \cdot \underbrace{\ln(p)}_{f}\, dp = \underbrace{2\frac{1}{6}p^6}_{g} \cdot \underbrace{\ln(p)}_{f} - \int \underbrace{\frac{2}{6}p^6}_{g} \cdot \underbrace{\frac{1}{p}}_{f'}\, dp$$

$$= \frac{1}{3}p^6 \ln(p) - \int \frac{1}{3}p^5\, dp =$$

$$= \frac{1}{3}p^6 \cdot \ln(p) - \frac{1}{3}\cdot\frac{1}{6}p^6 + C$$

$$= \underline{\frac{1}{3}p^6 \cdot \ln(p) - \frac{1}{18}p^6 + C}$$

#2

$$\int \underbrace{3x}_{f}\ \underbrace{\cos(x)}_{g'}\, dx = \underbrace{3x}_{f}\ \underbrace{\sin(x)}_{g} - \int \underbrace{3}_{f'} \cdot \underbrace{\sin(x)}_{g}\, dx$$

$$= 3x\sin(x) - 3(-\cos(x)) + C$$

$$= \underline{3x\sin(x) + 3\cos(x) + C}$$

#3

$$\int e^x \sin(x)\, dx$$

here we have to do integration by parts twice !!

$$\int \underbrace{e^x}_{f} \cdot \underbrace{\sin(x)}_{g'}\, dx = \underbrace{e^x}_{f}\underbrace{(-\cos(x))}_{g} - \int \underbrace{e^x}_{f'} \underbrace{(-\cos(x))}_{g}\, dx$$

$$= -e^x \cos(x) + \int e^x \cos(x)\, dx$$

apply again integration by parts

$$= -e^x \cos(x) + \left[\underbrace{e^x}_{f} \underbrace{\sin(x)}_{g} - \int \underbrace{e^x}_{f'} \cdot \underbrace{\sin(x)}_{g}\, dx \right]$$

Compare top and bottom

$$\int e^x \sin(x)\, dx = \cdots = -e^x \cos(x) + e^x \sin(x) - \underbrace{\int e^x \sin(x)\, dx}_{\text{move to the left}}$$

$$2\int e^x \sin(x)\, dx = e^x\big(\sin(x) - \cos(x)\big) + C$$

$$\therefore \quad \int e^x \sin(x)\, dx = \frac{1}{2} e^x \big(\sin(x) - \cos(x)\big) + C$$

$\boxed{\#4}$ $\int x^3 \cdot e^{-x^2/2}\, dx$

Let $\boxed{u = \frac{x^2}{2}}$ so $\frac{du}{dx} = \frac{1}{2} \cdot 2x$

OR $\boxed{du = x\, dx.}$ Notice that $\boxed{x^2 = 2u}$.

Thus the integral becomes

$$\int x^2 \cdot x \cdot e^{-x^2/2}\, dx = \int \underbrace{(2u)}_{f} \cdot \underbrace{e^{-u}}_{g'}\, du$$

$$= \underbrace{2u}_{f} \cdot (\underbrace{-e^{-u}}_{g}) - \int \underbrace{2}_{f'} (\underbrace{-e^{-u}}_{g})\, du$$

$$= -2u e^{-u} + 2\int e^{-u}\, du$$

$$= -2u e^{-u} + 2(-e^{-u}) + C$$

$$= -2u e^{-u} - 2e^{-u} + C$$

$$= -2e^{-u}(u+1) + C = -2e^{-\frac{x^2}{2}} \cdot \left(\frac{x^2}{2} + 1\right) + C$$

$$= \boxed{-e^{-x^2/2}(x^2+2) + C}$$

$\boxed{\#5}$

(a) $\int 2x \cdot \sin(x^2)\, dx$ set $u = x^2$

$\frac{du}{dx} = 2x$

so $du = 2x\,dx$

$= \int \sin(u)\, du$

$= -\cos(u) + C = \boxed{-\cos(x^2) + C}$

(b) $\int \underbrace{2x^2}_{f} \cdot \underbrace{\sin(x)}_{g'}\, dx$ = by parts

$= \underbrace{2x^2}_{f} \cdot (\underbrace{-\cos(x)}_{g}) - \int \underbrace{4x}_{f'} (\underbrace{-\cos(x)}_{g})\, dx$

$= -2x^2 \cos(x) + \int 4x \cos(x)\, dx$ = by parts again

$= -2x^2 \cos(x) + \left[\underbrace{4x}_{f} \underbrace{\sin(x)}_{g} - \int \underbrace{4}_{f'} \underbrace{\sin(x)}_{g}\, dx \right]$

$= \boxed{-2x^2 \cos(x) + 4x \sin(x) + 4 \cos(x) + C}$

$\boxed{\#6}$ $\int_1^5 x \cdot g'(x)\, dx = x \cdot g(x) \Big|_1^5 - \int_1^5 1 \cdot g(x)\, dx$

$= 5 \cdot g(5) - 1 \cdot g(1) - \int_1^5 g(x)\, dx = 5 \cdot 6 - (-3) - (-9)$

$= \boxed{42}$