$\boxed{\text{WORKSHEET \#5}}$

$\boxed{\#1}$

$$\int \underbrace{x}_{f} \cdot \underbrace{e^{-2x}}_{g'} dx = \underbrace{x}_{f} \cdot \underbrace{\left[-\frac{1}{2} e^{-2x}\right]}_{g} - \int \underbrace{1}_{f'} \cdot \underbrace{\left(-\frac{1}{2} e^{-2x}\right)}_{g} dx$$

$$= -\frac{1}{2} x e^{-2x} + \frac{1}{2} \int e^{-2x} dx =$$

$$= -\frac{1}{2} x e^{-2x} + \frac{1}{2}\left(-\frac{1}{2} e^{-2x}\right) + C$$

$$= \boxed{-\frac{1}{2} x e^{-2x} - \frac{1}{4} e^{-2x} + C}$$

$\boxed{\#2}$

$$\int_1^4 x\, f''(x)\, dx = \int_1^4 x \left(f'(x)\right)' dx =$$

$$= x f'(x) \Big|_1^4 - \int_1^4 1 \cdot f'(x) dx$$

$$= x f'(x) \Big|_1^4 - \int_1^4 f'(x) dx$$

$$= x f'(x) \Big|_1^4 - f(x) \Big|_1^4 =$$

$$= 4 \cdot f'(4) - 1 \cdot f'(1) - (f(4) - f(1))$$

$$= 4(-7) - 7 - (-2) + (-5) =$$

$$= -28 - 7 + 2 - 5 = \boxed{-38}$$

$\boxed{\#3}$ $\displaystyle\int \frac{4x+1}{(x-5)(x+2)}\,dx$

$$\frac{4x+1}{(x-5)(x+2)} = \frac{A}{x-5} + \frac{B}{x+2} = \frac{A(x+2) + B(x-5)}{(x-5)(x+2)}$$

Thus $\quad 4x+1 = A(x+2) + B(x-5)$

Use coverup method!

$\boxed{x=-2} \longrightarrow \quad 4(-2)+1 = A \cdot 0 + B(-2-5)$

so $\quad -7 = B(-7) \quad \boxed{B=1}$

$\boxed{x=5} \longrightarrow \quad 4(5)+1 = A(5+2) + B \cdot 0$

$21 = 7A \quad \boxed{A=3}$

Thus $\displaystyle\int \frac{4x+1}{x^2-3x-10}\,dx = \int \frac{4x+1}{(x-5)(x+2)}\,dx$

$$= \int \left(\frac{3}{x-5} + \frac{1}{x+2}\right) dx = 3\ln|x-5| + \ln|x+2| + C$$

$$= \boxed{\ln\left[\,|x-5|^3 \cdot |x+2|\,\right] + C}$$

$\boxed{\#4}$ $\displaystyle\int \frac{x^4+3}{x^2-4x+3}\,dx$ the rational function is <u>not</u> proper

Let's use the long division algorithm

$$\begin{array}{r|l}
 & x^2 + 4x + 13 \\
\hline
x^2-4x+3 & x^4 \qquad\qquad\qquad + 3 \\
 & x^4 - 4x^3 + 3x^2 \quad \text{subtract} \\
\cline{2-2}
 & 0 \quad 4x^3 - 3x^2 + 3 \\
 & \quad 4x^3 - 16x^2 + 12x \quad \text{subtract} \\
\cline{2-2}
 & \quad 0 \quad 13x^2 - 12x + 3 \\
 & \quad\quad 13x^2 - 52x + 39 \quad \text{subtract} \\
\cline{2-2}
 & \quad\quad 0 \quad 40x - 36
\end{array}$$

In other words

$$\boxed{x^4+3 = (x^2+4x+13)(x^2-4x+3) + 40x-36}$$

Thus

$$\int \frac{x^4+3}{x^2-4x+3}\,dx = \int \frac{(x^2+4x+13)(x^2-4x+3) + 40x-36}{x^2-4x+3}\,dx$$

$$= \int (x^2+4x+13)\,dx + \underline{\underline{\int \frac{40x-36}{(x-3)(x-1)}\,dx}}$$

$$\frac{40x-36}{(x-3)(x-1)} = \frac{A}{x-3} + \frac{B}{x-1} = \frac{A(x-1) + B(x-3)}{(x-3)(x-1)}$$

<u>So</u>: $40x-36 = A(x-1) + B(x-3)$

set $\boxed{x=1}$ $\longrightarrow$ $40-36 = A\cdot 0 + B(-2)$ $\therefore \boxed{B=-2}$

set $x=3$ $\longrightarrow$ $40\cdot 3-36 = A\cdot(3-1) + B\cdot 0$

$84 = 2A$ $\boxed{A = 42}$

$$\frac{40x-36}{(x-3)(x-1)} = \frac{42}{x-3} - \frac{2}{x-1}$$

Hence

$$\int \frac{x^4+3}{x^2-4x+3}\,dx = \int (x^2+4x+13)\,dx + \int \frac{42}{x-3}\,dx$$

$$-\int \frac{2}{x-1}\,dx$$

$$= \boxed{\frac{1}{3}x^3 + 2x^2 + 13x + 42\ln|x-3| - 2\ln|x-1| + C}$$