WORKSHEET #7

#1 Given $y = \tan^{-1} x$ we want to find $\frac{dy}{dx}$. For this, we write

$$\tan y = x$$

and apply $\frac{d}{dx}$ to both sides of the equation.

$$\frac{d}{dx}(\tan y) = \frac{d}{dx}(x)$$

apply chain rule

$$\frac{d}{dy}(\tan y) \cdot \frac{dy}{dx} = 1$$

$\Longleftrightarrow$

$$\sec^2 y \cdot \frac{dy}{dx} = 1$$

$\Longleftrightarrow$

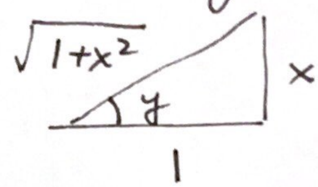
$$\frac{1}{\cos^2 y} \frac{dy}{dx} = 1$$

$\Longleftrightarrow$

$$\boxed{\frac{dy}{dx} = \cos^2 y}$$

Let's use right triangles trigonometry.

$\tan y = x \iff$



so $\cos y = \dfrac{1}{\sqrt{1+x^2}}$ and $\cos^2 y = \dfrac{1}{1+x^2}$

Thus

$$\boxed{\frac{dy}{dx} = \frac{d}{dx}\left(\tan^{-1} x\right) = \frac{1}{1+x^2}}$$

$\boxed{\#2}$

$$\int_{-1}^{+\infty} \frac{1}{(x+6)^{3/2}}\,dx = \text{ use substitution}$$

$u = x + 6$

$\dfrac{du}{dx} = 1$ or $du = dx$

$$= \int_{5}^{+\infty} \frac{1}{u^{3/2}}\,du = \lim_{b \to \infty} \int_{5}^{b} u^{-3/2}\,du$$

$$= \lim_{b \to +\infty} \left(\frac{1}{-\frac{3}{2}+1} u^{-\frac{3}{2}+1} \Bigg|_{5}^{b} \right)$$

$$= \lim_{b \to +\infty} \left(-2\,u^{-1/2} \Bigg|_{5}^{b} \right)$$

$$= \lim_{b\to\infty} \left(\frac{-2}{\sqrt{b}} - \left(\frac{-2}{\sqrt{5}}\right)\right) = \frac{2}{\sqrt{5}} - 2\underbrace{\lim_{b\to\infty} \frac{1}{\sqrt{b}}}_{0}$$

$$= \boxed{2/\sqrt{5}}$$

$\boxed{\#3}$ $\displaystyle\int_e^{+\infty} \frac{1}{x} \cdot \frac{1}{\ln x}\, dx$ = use substitution

$u = \ln x$

$\frac{du}{dx} = \frac{1}{x}$ or

$du = \frac{1}{x} dx$

$$= \int_1^{+\infty} \frac{1}{u}\, du$$

$$= \lim_{b\to+\infty} \int_1^b \frac{1}{u}\, du = \lim_{b\to+\infty} \left(\ln|u| \Big|_1^b \right)$$

$$= \lim_{b\to+\infty} \left[\ln(b) - \underbrace{\ln(1)}_{0} \right] =$$

$$= \lim_{b\to+\infty} \ln(b) = +\infty$$

Hence the given integral does not converge

$\boxed{\#4}$

$$\int_1^e \frac{1}{x}\cdot\frac{1}{\sqrt{\ln x}}\,dx =$$ use the substitution

$u = \ln x$

$\frac{du}{dx} = \frac{1}{x}$ or

$du = \frac{1}{x}\,dx$

$$= \int_0^1 \frac{1}{\sqrt{u}}\,du$$ this integral is improper as $\frac{1}{\sqrt{u}} \to +\infty$ as $u \to 0^+$

$$= \lim_{a\to 0^+} \int_a^1 u^{-1/2}\,du$$

$$= \lim_{a\to 0^+} \left(\frac{1}{1/2} u^{1/2} \Big|_a^1 \right) =$$

$$= \lim_{a\to 0^+} \left(2\sqrt{u} \Big|_a^1 \right) =$$

$$= \lim_{a\to 0^+} \left(2\sqrt{1} - 2\sqrt{a} \right) =$$

$$= 2 - 2\cdot \underbrace{\lim_{a\to 0^+} \sqrt{a}}_{0} = \boxed{2}$$