$\boxed{\text{WORKSHEET \#9}}$

$\boxed{\#4}$ $$\frac{dy}{dx} + 0.3xy = 3x \qquad y(0) = 5$$

Rewrite as $$\frac{dy}{dx} = 3x - 0.3xy \qquad (*)$$

$$\frac{dy}{dx} = -0.3x(y-10) \qquad (*)$$

separate variables and integrate

$$\int \frac{1}{y-10}\, dy = \int -0.3x\, dx$$

it is better to have the coefficient of $y$ equal to 1; that's why I factored $-0.3x$ out in $(*)$

$$\ln|y-10| = -0.15x^2 + C$$

Take exponential of both sides

$$e^{\ln|y-10|} = e^{-0.15x^2 + C}$$

$$|y-10| = e^{c} \cdot e^{-0.15x^2}$$

Get rid of absolute value

$$y-10 = \underbrace{\pm e^{c}}_{\substack{\text{rename constant} \\ A}} \cdot e^{-0.15x^2}$$

$$y = 10 + Ae^{-0.15x^2}$$

Since $y(0) = 5$

$$5 = 10 + A \cdot \underbrace{e^{0}}_{=1} \qquad \boxed{A = -5}$$

Thus $\boxed{y = 10 - 5e^{-0.15x^2}}$

$\boxed{\#2}$ $\quad \dfrac{dy}{dx} = \dfrac{\ln x}{xy} \qquad y(1) = 5$

Rewrite as

$$y\,dy = \frac{1}{x} \cdot \ln x \, dx$$

and integrate with respect to x and y respectively.

$$\int y\,dy = \int \frac{1}{x}\cdot \ln x \, dx$$

use the substitution $u = \ln x$

$du = \frac{1}{x}\,dx$

$$\int y\,dy = \int u \cdot du$$

$$\frac{1}{2}y^2 = \frac{1}{2}u^2 + C$$

$$y^2 = u^2 + \underbrace{2C}_{\text{call it } \tilde{C} \text{ a new constant}}$$

So $y^2 = (\ln x)^2 + \tilde{C}$

Since $y(1) = 5$ we have

$$25 = 5^2 = (\ln(\cancel{1}))^2 + \tilde{C} \quad \therefore \tilde{C} = 25$$

Thus

$$\boxed{y = \sqrt{(\ln(x))^2 + 25}}$$

since $y(1)=5$ is positive we could not take the negative solution when we took the $\sqrt{\cdot}$.