# MA 138 Worksheet #1

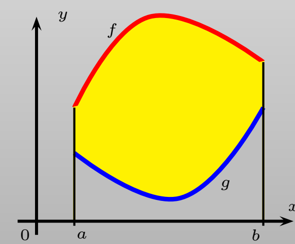
Syllabus & Section 6.3
1/9/24

**1** Syllabus discussion.

**Area between curves**

Assume $f$ and $g$ are continuous and $f(x) \geq g(x)$ for all $x$ in $[a, b]$. The area $A$ of the region bounded by the curves $y = f(x)$, $y = g(x)$, and the lines $x = a$, $x = b$, is

$$A = \int_a^b [f(x) - g(x)]\, dx.$$




**2** Find the area enclosed between $f(x) = 0.9x^2 + 7$ and $g(x) = x$ from $x = -4$ to $x = 8$.

**3** Sketch the region enclosed by $x + y^2 = 20$ and $x + y = 0$. Decide whether to integrate with respect to $x$ or $y$, and then find the area of the region.

**Average of a continuous function over an interval**

The average value of a continuous function $f$ defined on the interval $[a, b]$ is given by

$$f_{\text{avg}} = \frac{1}{b-a} \int_a^b f(x)\, dx.$$

Moreover, by the Fundamental Theorem of Calculus there exists a number $c \in (a, b)$ such that

$$f(c)(b-a) = \int_a^b f(x)\, dx.$$

That is, when $f$ is continuous, there exists a number $c$ such that $f(c) = f_{\text{avg}}$. If $f$ is a continuous, positive valued function, $f_{\text{avg}}$ is that number such that the rectangle with base $[a, b]$ and height $f_{\text{avg}}$ has the same area as the region underneath the graph of $f$ from $a$ to $b$.

**4** Find the average value of the function $f(x) = \dfrac{-8}{x}$ on the interval $[1, 4]$.

**5** In a certain city the temperature (in degrees Fahrenheit) $t$ hours after 9 am was approximated by the function $T(t) = 50 + 6 \sin\left(\dfrac{\pi t}{12}\right)$.

(a) Determine the temperature at 9 am.

(b) Determine the temperature at 3 pm.

(c) Find the average temperature during the period from 9 am to 9 pm.

**Cumulative change**

Given the initial value problem $\frac{dN}{dt} = f(t) \qquad N(a) = N_a$ has solution

$$\boxed{N(t) - N(a) = \int_a^t f(u)\,du = \int_a^t \frac{dN}{du}\,du}.$$

In other words,

$$\left\{ \begin{array}{c} \text{cumulative change} \\ \text{on the interval } [a,t] \end{array} \right\} = \int_a^t \left\{ \begin{array}{c} \text{instantaneous rate of} \\ \text{change at time } u \end{array} \right\} du$$

**6** Recall that the acceleration $a(t)$ of a particle moving along a straight line is the instantaneous rate of change of the velocity $v(t)$; that is, $a(t) = \frac{d}{dt}v(t)$. Assume that $a(t) = 32$ ft/s$^2$.

(a) Express the cumulative change in velocity during the interval [0,t] as a definite integral, and compute the integral.

(b) Given that the initial velocity of the object is 5 ft/s, find a formula for $v(t)$.

(c) Now, given your response to part (b), find the cumulative change in position during the first 10 seconds of free-fall.