MA 138 Worksheet #10

Section 8.1

2/8/24

1 Find the particular solution of the differential equation $\dfrac{dy}{dx} = (x-2)e^{-2y}$ satisfying the initial condition $y(2) = \ln(2)$.

Pure-Time and Autonomous Differential Equations (Section 8.1)

First-order separable differential equations (DEs), that is DEs of the form $\dfrac{dy}{dx} = f(x)g(y)$, include two important special cases:

- **pure-time differential equations**: $\dfrac{dy}{dx} = f(x)$ [i.e., $g(y) \equiv 1$]

  The solution that satisfies this DE with initial condition $y(x_0) = x_0$ can be written as

  $$\boxed{y(x) = y_0 + \int_{x_0}^{x} f(u)\,du}\,.$$

  (It may be helpful to recall the discussion of cumulative change in Section 6.3!)

- **autonomous differential equations**: $\dfrac{dy}{dx} = g(y)$ [i.e., $f(x) \equiv 1$]

  To find the general solution of this DE we integrate $\boxed{\displaystyle\int \frac{1}{g(y)}dy = \int dx}\,.$

  (DEs of this form are frequently used in biological models.)

2 Solve each pure-time differential equation:

(a) $\dfrac{dy}{dx} = e^{-3x}$ where $y_0 = 10$ for $x_0 = 0$;

(b) $\dfrac{dx}{dt} = \dfrac{1}{1-t}$ where $x(0) = 2$;

(c) $\dfrac{ds}{dt} = \sqrt{3t+1}$ where $s(0) = 1$.

3 Solve the given autonomous differential equations:

(a) $\dfrac{dx}{dt} = -2x$ where $x(1) = 5$;

(b) $\dfrac{dh}{dt} = 2h + 1$ where $h(0) = 4$;

(c) $\dfrac{dy}{dx} = 2y(3-y)$ where $y_0 = -3$ for $x_0 = 0$.

[**hint:** use the partial-fraction method to solve the DE]