MA 138 Worksheet #11

Section 8.1

2/13/24

1 Suppose that a population, whose size at time t is given by $N(t)$, grows according to the differential equation $\dfrac{dN}{dt} = \dfrac{1}{100}N^2$, with $N(0) = 10$.

(a) Solve the differential equation.

(b) Graph $N(t)$ as a function of t for $0 \leq t < 10$. What happens as $t \longrightarrow 10$? Explain in words what this means.

2 Denote by $L(t)$ the length of a certain fish at time t, and assume that this fish grows according to the von Bertalanffy equation $\dfrac{dL}{dt} = k(L_\infty - L(t))$ with $L(0) = 1$. Assume further that k and L_∞ are positive constants.

A study showed that the asymptotic length is equal to 123 inches and that it takes this fish 27 months to reach half its asymptotic length.

(a) Use this information to determine the constants k and L_∞.

(b) Determine the length of the fish after 10 months.

(c) How long will it take until the fish reaches 90% of its asymptotic length?

3 Suppose that news spreads through a city of fixed size of 900,000 people at a time rate proportional to the number of people who have not heard the news.

(a) Formulate a differential equation for $y(t)$, the number of people who have heard the news t days after it has happened. No one has heard the news at first, so you may assume that $y(0) = 0$.

(b) Six days after a scandal in City Hall was reported, a poll showed that 450,000 people have heard the news. Using this information and the differential equation, solve for the number of people who have heard the news after t days.

4 Biologists stocked a lake with 500 fish and estimated the carrying capacity to be 9,500. The number of fish tripled in the first year.

(a) Assuming that the size of the fish population satisfies the logistic differential equation $\dfrac{dP}{dt} = kP\left(1 - \dfrac{P}{K}\right)$, determine the constant k, and then solve the equation to find an expression for the size of the population after t years.

(b) How long will it take for the population to increase to 4,750 (half of the carrying capacity)?