MA 138 Worksheet #14

Sections 8.2 & 9.1

2/22/24

1 Suppose that $N(t)$ denotes the size of a population at time t. The population evolves according to the logistic equation, but, in addition, predation reduces the size of the population so that the rate of change is given by $\dfrac{dN}{dt} = N\left(1 - \dfrac{N}{50}\right) - \dfrac{9N}{5+N}$.

(a) Set $g(N) = N\left(1 - \dfrac{N}{50}\right) - \dfrac{9N}{5+N}$ and graph $g(N)$.

(b) Find all equilibria of the differential equation.

(c) Use the method of eigenvalues to determine the stability of the equilibria you found in (b).

(d) Use your graph from before to determine the stability of the equilibria you found analytically.

2 Suppose that a fish population evolves according to the logistics equation and that a fixed number of fish per unit time are removed. That is, $\dfrac{dN}{dt} = rN\left(1 - \dfrac{N}{K}\right) - H$.

Assume that $r = 2$ and $K = 1000$.

(a) Find possible equilibria, and discuss their stability when $H = 100$.

(a) What is the maximal harvesting rate that maintains a positive population size?

Solving Systems of Linear Equations

To solve systems of linear equations we can use two methods:

- the substitution method
- the elimination method.

- In the **substitution method** we follow the procedure outlined below. This method works for systems of linear equations in few variables.
  1. **Solve for One Variable:** Choose one equation and solve for one variable in terms of the other variable.
  2. **Substitute:** Substitute the expression you found in Step 1 into the other equation to get an equation in one variable, then solve for that variable.
  3. **Back-Substitute:** Substitute the value you found in Step 2 back into the expression found in Step 1 to solve for the remaining variable.
- To solve large systems of m linear equations in n variables, we need to develop a more efficient method: the **elimination method**.
  - The basic strategy is to transform the system of linear equations into an equivalent system of equations that has the same solutions as the original, but a much simpler form.
  - In a nutshell, in the elimination method we try to combine the equations using sums or differences to eliminate one of the variables.

Number of Solutions of a Linear System in Two Variables

For a system of linear equations in two variables, exactly one of the following is true:

1. The system has exactly one solution.
2. The system has no solution.
3. The system has infinitely many solutions.

3 Solve the system using substitution first. Then repeat the problem using elimination.

$$\begin{cases} -8x - 7y = 44 \\ 7x + 4y = -30 \end{cases}$$

4 If the following system is consistent, then k <u>cannot</u> equal what value?

$$\begin{cases} -12x + 15y = 6 \\ 20x + ky = 12 \end{cases}$$