MA 138 Worksheet #15

Section 9.1

2/27/24

General Setup

A system of m equations in n variables can be written in the form

$$\begin{cases} a_{11}x_1 & + & a_{12}x_2 & + \cdots + & a_{1n}x_n & = & b_1 \\ a_{21}x_1 & + & a_{22}x_2 & + \cdots + & a_{2n}x_n & = & b_2 \\ \vdots & & \vdots & \ddots & \vdots & & \vdots \\ a_{m1}x_1 & + & a_{m2}x_2 & + \cdots + & a_{mn}x_n & = & b_m \end{cases}$$

- The variables are now $x_1, x_2, \ldots, x_n$.
- The coefficients a_{ij} on the left-hand side have two subscripts. The first subscript (that is, 'i') indicates the equation, and the second subscript (that is 'j') indicates to which variable a_{ij} corresponds to.

Gaussian Elimination Method

The main idea in the elimination method is that we transform the given system of linear equations into an equivalent one ($\equiv$ the new system has the same solutions as the old one).

To do so, we will use the following three basic operations:

1. multiplying an equation by a nonzero constant;
2. adding one equation to another;
3. rearranging the order of the equations.

This method is also called **Gaussian elimination method**.

The three basic operations in the Gaussian elimination method make changes only to the coefficients of the variables. Thus we will work on the **augmented matrix**:

$$\begin{array}{r} \\ \text{1st row} \longrightarrow \\ \text{2nd row} \longrightarrow \\ \\ m\text{th row} \longrightarrow \end{array} \begin{array}{c} \begin{array}{cccc} \text{1st column} \downarrow & \text{2nd column} \downarrow & & n\text{th column} \downarrow \end{array} \\ \left[\begin{array}{cccc|c} a_{11} & a_{12} & \cdots & a_{1n} & b_1 \\ a_{21} & a_{22} & \cdots & a_{2n} & b_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} & b_m \end{array}\right] \end{array}$$

Remarks

- Ideally the systems that we would like to encounter have the same number of equations as variables. This need not be the case.
- A system with fewer equations than variables is said to be **underdetermined**. They frequently have infinitely many solutions.
- A system with more equations than variables is said to be **overdetermined**. They frequently are inconsistent.
- A useful software to perform Gaussian Elimination can be found here:

  `https://www.zweigmedia.com/RealWorld/tutorialsf1/scriptpivot2.html`

1 Find k and h so that there are an infinite number of solutions to the following system.

$$\begin{cases} 9x + 8y = h \\ -4x + ky = -2 \end{cases}$$

2 Find the solution of the system of linear equations

$$\begin{cases} 3x_1 + 5x_2 - x_3 = 10 \\ 2x_1 - x_2 + 3x_3 = 9 \\ 4x_1 + 2x_2 - 3x_3 = -1 \end{cases}$$

(practice with the software mentioned above.)

3 Determine the value of k for which the following system has no solutions.

$$\begin{cases} x + y + 3z = -3 \\ x + 2y - 3z = -1 \\ 5x + 12y + kz = 0 \end{cases}$$

4 Determine c such that

$$\begin{cases} 2x - 3y = 5 \\ 4x - 6y = c \end{cases}$$

the above system has:

(a) infinitely many solutions

(b) no solutions

(c) Is it possible to choose a number for c so that the system has exactly one solution? Explain your answer.