MA 138 Worksheet #17

Review for Exam 2 & Section 9.2

3/5/24

1 Exam 2 has the same structure as the previous exam. It covers the topics from Section 8.1 to Section 9.2 (Lectures 11 through 22).

2 Make sure to be familiar with the types of problems discussed in the lectures, homework assignments, and recitation worksheets:

- Solving differential equations (Section 8.1 — Lectures 11, 12, and 14):
  - what it means to be a solutions of a differential equation;
  - separable differential equations;
  - pure-time and autonomous differential equations;
  - differential equations of interest in the Life Sciences (exponential, logistic, Von Bertalanffy differential equations).
- Direction fields and SAGE (Handout — Lectures 15 and 16)
- Equilibria and their stability (Section 8.2 — Lectures 17 and 18):
  - equilibria of an autonomous differential equation: locally stable vs unstable equilibrium;
  - analytic approach to stability (Stability Criterion);
  - graphical approach to stability.
- Linear systems (Section 9.1 — Lectures 19 and 20):
  - number of solutions of a linear system (one, none, infinitely many);
  - substitution vs elimination method;
  - augmented matrix;
  - Gaussian elimination process.
- Matrices (Section 9.2 — Lectures 21 and 22):
  - basic matrix operations (addition, multiplication by scalars; transpose);
  - matrix multiplication;
  - inverse of matrices;
  - application to solving systems of n linear equations in n variables.

3 Use the old exams as a guide to possible questions. The previous quizzes can also serve as a guide. Check the solutions provided online to see where you made mistakes in the previous quizzes.

4 Show that the inverse of $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$ is $A^{-1} = \begin{bmatrix} 3/4 & 1/2 & 1/4 \\ 1/2 & 1 & 1/2 \\ 1/4 & 1/2 & 3/4 \end{bmatrix}$

5 Suppose $A = \begin{bmatrix} 1 & 2 \\ 3 & -5 \end{bmatrix}$ and $B = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$. Find $X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ such that $AX = B$ by:

(a) solving the associated system of linear equations;

(b) using the inverse of A.

6 Find the inverse (if it exists) of $A = \begin{bmatrix} 5 & 2 \\ -7 & -3 \end{bmatrix}$.

7 Find the inverse (if it exists) of $B = \begin{bmatrix} -1 & 0 & -1 \\ 2 & 1 & 1 \\ -1 & 1 & 2 \end{bmatrix}$.

8 Find the inverse (if it exists) of $C = \begin{bmatrix} 3 & 1 & -1 \\ 2 & -1 & 2 \\ 2 & 1 & -2 \end{bmatrix}$.

9 Let $A = \begin{bmatrix} -1 & 1 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 0 \\ 3 & 2 \end{bmatrix}$. Show that $(AB)^{-1} = B^{-1}A^{-1}$.

10 Let $AB = \begin{bmatrix} 0 & 1 \\ 2 & -1 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 4 & -1 \\ 8 & -1 \end{bmatrix}$. Find B.