MA 138 Worksheet #18

Section 9.3

3/7/24

1 Let $\mathbf{u} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} -1 \\ -3 \end{bmatrix}$, and $\mathbf{w} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$ be vectors in $\mathbb{R}^2$.

Find $-1/2\,\mathbf{u}$, $\mathbf{u} + \mathbf{v}$, $\mathbf{v} - \mathbf{w}$, $-\mathbf{v}$, and illustrate the results graphically.

2 Use a rotation matrix to rotate the vector $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$ counterclockwise by the angle $\pi/3$.

3 Match each linear transformation (labeled A. through F.) with its corresponding matrix (labeled (a) through (f)):

(a) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

A. Indentity transformation

(b) $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

B. Reflection in the x-axis

C. Reflection in the origin

(c) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

D. Projection onto the y-axis

(d) $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

E. Contraction by a factor of 2

F. Rotation through an angle of 90 degrees in the counterclockwise direction

(e) $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

(f) $\begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$

4 Compute the determinant of the rotation matrix in #2. Compute the determinant of any rotation matrix.

5 Determine if $\mathbf{v}$ is an eigenvector of the matrix A. If so, find the corresponding eigenvalue λ.

(a) $A = \begin{bmatrix} 68 & 30 \\ -150 & -67 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 2 \\ 7 \end{bmatrix}$

(b) $A = \begin{bmatrix} 22 & -9 \\ 30 & -11 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} -1 \\ -2 \end{bmatrix}$

(c) $A = \begin{bmatrix} 4 & -8 \\ 0 & -4 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$