

MA 138 Worksheet #20

Section 9.3 & Least Squares Approximations

3/21/24

1 (Complex eigenvalues) Consider the matrix $A = \begin{bmatrix} 3 & -2 \\ 4 & -1 \end{bmatrix}$.

- (a) Find its eigenvalues.
- (b) Find the eigenvectors associated with the eigenvalues from part (a).

2 Find a 2×2 matrix A such that $\begin{bmatrix} -2 \\ 4 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 3 \end{bmatrix}$ are eigenvectors of A with eigenvalues 3 and -9 , respectively.

3 Let $A = \begin{bmatrix} 5 & 7 \\ -2 & -4 \end{bmatrix}$. Find $A^{20} \begin{bmatrix} -3 \\ -2 \end{bmatrix}$.

4 Find the least squares solution $\mathbf{x}^* = \begin{bmatrix} x_1^* \\ x_2^* \end{bmatrix}$ of the system $\begin{bmatrix} 1 & -1 \\ -1 & 1 \\ 3 & 5 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 10 \\ -2 \\ -22 \end{bmatrix}$.

5 During the summer months Terry makes and sells necklaces on the beach. Terry notices that if he lowers the price, he can sell more necklaces, and if he raises the price than he sells fewer necklaces. The table below shows how the number n of necklaces sold in one day depends on the price p (in dollars).

Price	Number of necklaces sold
8	34
11	20
15	13

Find a linear function of the form $n = c_0 + c_1p$ that best fits these data, using least squares.