

# MA 138 Worksheet #22

Sections 10.1 & 10.2

3/28/24

1. Let  $f_a(x, y) = ax^2 + y^2$  for  $(x, y) \in \mathbb{R}$ , where  $a$  is a positive constant.

- (a) Assume that  $a = 1$  and describe the level curves of  $f_1(x, y) = x^2 + y^2$ . The graph of  $f_1(x, y)$  intersects both the  $x$ - $z$  and  $y$ - $z$  planes; show that these two curves of intersection are parabolas.
- (b) Assume that  $a = 4$ . Then  $f_4(x, y) = 4x^2 + y^2$  and the level curves satisfy  $4x^2 + y^2 = c$ . Use a graphing calculator to sketch the level curves for  $c = 0, 1, 2, 3, 4$ . These curves are ellipses. Find the curves of intersection of  $f_4(x, y)$  with  $x$ - $z$  and the  $y$ - $z$  planes.
- (c) Repeat (b) for  $a = 1/4$ .
- (d) Explain in words how the surfaces of  $f_a(x, y)$  change when  $a$  changes.

2. Find the limit  $\lim_{(x,y) \rightarrow (45,9)} xy \cos(x - 5y)$ .

3. Find the limit  $\lim_{(x,y) \rightarrow (0,0)} \frac{(9x + y)^2}{81x^2 + y^2}$ .

- (a) Along the  $x$ -axis;
- (b) Along the  $y$ -axis;
- (c) Along the line  $y = x$ ;
- (d) Along the line  $y = mx$ ;
- (e) What is the limit?