MA 138 Worksheet #24

Sections 10.3 & 10.4

4/4/24

1. Find the partial derivatives $f_x(x,y)$, $f_y(x,y)$, $f_{xy}(x,y)$ and $f_{yx}(x,y)$ of $f(x,y) = xye^{-5y}$.

2. Given $f(x,y) = \ln(x^3 + y^5)$, compute $\dfrac{\partial f}{\partial x}$ and $\dfrac{\partial f}{\partial y}$.

3. On a hot day, extreme humidity makes us think the temperature is higher than it really is, whereas in very dry air we perceive the temperature to be lower than the thermometer indicates. The National Weather Service has devised the *heat index* to describe the combined effects of temperature and humidity. The heat index I is the perceived air temperature when the actual temperature is T and the relative humidity is H. So I is a function of T and H and we can write $I = f(T,H)$. The following table of values of I is an excerpt from a table compiled by the National Weather Service.

		Relative humidity H (%)								
		50	**55**	**60**	**65**	**70**	**75**	**80**	**85**	**90**
Actual temperature T (°F)	**90**	96	98	100	103	106	109	112	115	119
	92	100	103	105	108	112	115	119	123	128
	94	104	107	111	114	118	122	127	132	137
	96	109	113	116	121	125	130	135	141	146
	98	114	118	123	127	133	138	144	150	157
	100	119	124	129	135	141	147	154	161	168

Estimate the rate of change of the heat index with respect to the temperature, that is $\dfrac{\partial f}{\partial T}$, when the temperature is 96°F and the humidity is 70%

4. Holling (1959) derived an expression for the number of prey items P_e eaten by a predator during an interval T as a function of prey density N and the handling time T_h of each prey item:

$$P_e = f(N,T,T_h) = \frac{aNT}{1 + aT_hN}.$$

Here, a is a positive constant called the predator attack rate. Find $\dfrac{\partial f}{\partial T}$, $\dfrac{\partial f}{\partial N}$, and $\dfrac{\partial f}{\partial T_h}$.

5. Find the equation of the tangent plane to the surface $z = 9x^5 + 7y^5 + 2xy$ at the point $(1, -3, -1698)$.