MA 138 Worksheet #25

Review for Exam 3 & Section 10.4

4/9/24

1. Exam 3 has the same structure as the previous exam. It covers the topics from Section 9.3 to Section 10.4 (Lectures 23 through 35).

2. Make sure to be familiar with the types of problems discussed in the lectures, homework assignments, and recitation worksheets:

- Linear Maps, Eigenvectors and Eigenvalues (Section 9.3 — Lectures 23, 24, and 26):
  - vectors and operations on vectors;
  - simple linear maps (reflections, rotations, shear maps);
  - properties of linear maps;
  - eigenvalues and eigenvectors;
  - characteristic equation and relation between the eigenvalues and the trace and determinant of a 2×2 matrix.
- Fibonacci's numbers, a population model, and powers of matrices (Handout — Lectures 26 and 27)
  - how to compute a large power of a matrix applied to a vector using eigenvalues and eigenvectors.
- Curve Fitting-Least Square Approximation (Handout — Lectures 28 and 29):
  - how to compute the best approximate solution (least squares solution) of an overdetermined system of linear equations.
- Functions of Two or More Independent Variables (Section 10.1 — Lecture 30):
  - domain and graph of a function of two variables;
  - level curves and contour lines of a function of two variables.
- Limits and Continuity (Section 10.2 — Lecture 31):
  - computation of limits of functions of two variables;
  - how to determine whether a limit at a point does not exist by choosing different paths through the point.
- Partial Derivatives (Section 10.3 — Lectures 32 and 33):
  - geometric interpretation of partial derivatives;
  - higher partial derivatives and the mixed derivative theorem.
- Tangent Planes, Differentiability, and Linearization (Section 10.4 — Lectures 34 and 35):
  - equation of the tangent plane to the graph of a function of two variables at a point;
  - linearization of a function of two variables at a point.

3. Use the old exams as a guide to possible questions. The previous quizzes can also serve as a guide. Check the solutions provided online to see where you made mistakes in the previous quizzes.

4. Find the equation of the tangent plane to the surface at the indicated point.

(a) $f(x,y) = e^{x-y}$ at $(1,-1,e^2)$;

(b) $f(x,y) = xy$ at $x_0 = -1, y_0 = -2$.

5. Linear Transformations.

(a) Give a geometric interpretation of the following matrices:

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \qquad C = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

(b) Use a rotation matrix to rotate the vector $\begin{bmatrix} 4 \\ -1 \end{bmatrix}$ counterclockwise by the angle $\pi/3$.

6. Eigenvalues/Eigenvectors. Find the eigenvalues and corresponding eigenvectors for each matrix:

$$A = \begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 1 \\ 2 & 3 \end{bmatrix}$$

7. Powers of Matrices. Compute the following without using a calculator:

(a) Let $A = \begin{bmatrix} -1 & 0 \\ 3 & 1 \end{bmatrix}$. Find $A^{15} \begin{bmatrix} 2 \\ 0 \end{bmatrix}$.

(b) Let $B = \begin{bmatrix} 4 & -3 \\ 2 & -1 \end{bmatrix}$. Find $A^{30} \begin{bmatrix} -4 \\ -2 \end{bmatrix}$.

8. Least Squares Approximation. Fit a linear function of the form $y = mx + b$ to the data points $(-9,-57), (0,3), (9,51)$ using least squares.

9. Domain/Range/Contour Lines. Find the largest possible domain and the corresponding range of each function. Determine the equation of the level curves.

$$f(x,y) = x^2 + y^2 \qquad g(x,y) = \ln(y - x^2) \qquad h(x,y) = \frac{x-y}{x+y}$$

9. Limits and Continuity. Determine the following limits:

$$\lim_{(x,y)\longrightarrow(0,2)} 4xy^2 - \frac{x+1}{y} \qquad \lim_{(x,y)\longrightarrow(0,0)} \frac{3xy}{x^2+y^3} \qquad \lim_{(x,y)\longrightarrow(0,0)} \frac{2xy}{x^3+xy}$$

10. Partial Derivatives. Find the indicated partial derivatives.

(a) $f(x,y) = x^2y + xy^2$; f_y and f_{yx}.

(b) $f(x,y) = \ln(3x^2 - xy)$; f_y.

(c) $f(u,v) = e^{u^2/2}\ln(u+v)$; f_u.