MA 138 Worksheet #28

Sections 11.1 & 11.2

4/18/24

1. Find the solution of the system of differential equations

$$\begin{cases} x' = & y \\ y' = & 4x + 3y \end{cases}$$

if $x(0) = 0$ and $y(0) = 5$.

2. Solve the following differential equation:

$$y'' - 3y' - 10y = 0$$

where $y = 1$, $y' = 10$ at $x = 0$.

3. John opens a bank account with an initial balance of 500 dollars. Let $b(t)$ be the balance in the account at time t. Thus $b(0) = 500$. The bank is paying interest at a continuous rate of 6% per year. John makes deposits into the account at a continuous rate of $s(t)$ dollars per year. Suppose that $s(0) = 500$ and that $s(t)$ is increasing at a continuous rate of 4% per year (John can save more as his income goes up over time).

   (a) Set up a linear system of the form:

   $$\begin{cases} \dfrac{db}{dt} = & m_{11}b + m_{12}s \\[2mm] \dfrac{ds}{dt} = & m_{21}b + m_{22}s \end{cases}$$

   for appropriate choices of m_{ij} with $i, j = 1, 2$.

   (b) Solve the initial value problem and find $b(t)$ and $s(t)$.

4. Suppose that a drug is administered to a person in a single dose, and assume that the drug does not accumulate in body tissue, but is excreted through urine. Denote the amount of drug in the body at time t by $x_1(t)$ and in the urine at time t by $x_2(t)$. If $x_1(0) = 4$ mg and $x_2(0) = 0$, find $x_1(t)$ and $x_2(t)$ if

   $$\frac{dx_1}{dt} = -0.3x_1(t).$$