MA 138 Worksheet #5

Sections 7.2 & 7.3

1/23/24

Rule for Integration by Parts

If $f(x)$ and $g(x)$ are differentiable functions, then

$$\int f(x)g'(x)\,dx = f(x)g(x) - \int f'(x)g(x)\,dx;$$

$$\int_a^b f(x)g'(x)\,dx = f(x)g(x)\Big]_a^b - \int_a^b f'(x)g(x)\,dx.$$

1 Use integration by parts to evaluate the indefinite integral: $\displaystyle\int xe^{-2x}\,dx.$

2 Suppose that $f(1) = -5,\ \ f(4) = -2,\ \ f'(1) = 7,\ \ f'(4) = -7,$ and f'' is continuous. Find the value of the definite integral $\displaystyle\int_1^4 x\,f''(x)\,dx.$

Partial Fraction Decomposition (case of distinct linear factors)

Suppose $f(x)$ is a **proper rational function** written as $P(x)/Q(x)$;
that is, $P(x)$ and $Q(x)$ are polynomials with $\deg P(x) < \deg Q(x)$.
Further, suppose that $Q(x)$ is a product of m distinct linear factors. $Q(x)$ is thus of the form:

$$Q(x) = a(x-\alpha_1)(x-\alpha_2)\cdots(x-\alpha_m)$$

where $\alpha_1, \alpha_2, \ldots, \alpha_m$ are the m distinct roots of $Q(x)$.
The rational function can then be written as

$$\frac{P(x)}{Q(x)} = \frac{1}{a}\left[\frac{A_1}{x-\alpha_1} + \frac{A_2}{x-\alpha_2} + \cdots + \frac{A_m}{x-\alpha_m}\right],$$

where $A_1, A_2, \ldots, A_m$ are appropriate constants.

3 Evaluate $\displaystyle\int \frac{4x+1}{x^2-3x-10}\,dx$ by using a partial fraction decomposition.

4 Follow the steps below to evaluate the integral: $\displaystyle\int \frac{x^4+3}{x^2-4x+3}\,dx.$

- (a) Use polynomial long division to write the function $\dfrac{x^4+3}{x^2-4x+3}$ as the sum of a polynomial and a *proper* rational function;
- (b) Find the partial fraction decomposition of the proper rational function obtained in part (a).
- (c) Evaluate the integral: $\displaystyle\int \frac{x^4+3}{x^2-4x+3}\,dx.$