MA 138 Worksheet #6

Sections 7.3

1/25/24

1 Make a substitution to express the integrand as a proper rational function and then evaluate the integral using a partial fraction decomposition $\displaystyle\int \frac{-e^{2x}}{e^{2x}+3e^x+2}\,dx.$

2 Evaluate $\displaystyle\int \frac{x+2}{x^3-x}\,dx$ by using a partial fraction decomposition.

Partial Fraction Decomposition (case of repeated linear factors)

$Q(x)$ is a product of m distinct linear factors to various powers. $Q(x)$ is thus of the form

$$Q(x) = a(x-\alpha_1)^{n_1}(x-\alpha_2)^{n_2}\cdots(x-\alpha_m)^{n_m}$$

where $\alpha_1, \alpha_2, \ldots, \alpha_m$ are the m distinct roots of $Q(x)$ and $n_1, n_2, \ldots, n_m$ are positive integers such that $n_1 + n_2 + \cdots + n_m = \deg Q(x)$.
The rational function can then be written as

$$\frac{P(x)}{Q(x)} = \frac{1}{a}\left[\sum_{i=1}^{m} \frac{A_{i,1}}{x-\alpha_i} + \frac{A_{i,2}}{(x-\alpha_i)^2} + \cdots + \frac{A_{i,n_i}}{(x-\alpha_i)^{n_i}}\right].$$

3 Evaluate $\displaystyle\int_3^8 \frac{6x}{x^2+4x+4}\,dx$ by using a partial fraction decomposition.

4 Evaluate $\displaystyle\int \frac{9x^2}{(x+1)^3}\,dx$ by using a partial fraction decomposition.

Partial Fraction Decomposition (case of irreducible quadratic factors)

If the irreducible quadratic factor ax^2+bx+c is contained n times in the factorization of the denominator of a proper rational function, then the partial-fraction decomposition contains terms of the form

$$\frac{B_1x+C_1}{ax^2+bx+c} + \frac{B_2x+C_2}{(ax^2+bx+c)^2} + \cdots + \frac{B_nx+C_n}{(ax^2+bx+c)^n}.$$

5 Write out the partial fraction decomposition for the function $\displaystyle\frac{x^2+3x-10}{(x^2+4x+6)^2(x^2-1)(x+1)}.$

Caution

Integrating rational functions that involve irreducible quadratic factors is really difficult, and is beyond the scope of this class. So we will not focus on those integrals.