MA 138 Worksheet #7

Sections 7.3 & 7.4

1/30/24

Caution

Integrating rational functions that involve irreducible quadratic factors in the denominator is really difficult, and is beyond the scope of this class. So we will not focus on those integrals. It suffices to say that integrating rational functions with irreducible quadratic factors in the denominator involve the following basic integral

$$\int \frac{1}{1+x^2}\,dx = \tan^{-1} x + C,$$

and completion of the square to reduce to the above integral.

1 Show that $\dfrac{d}{dx}\tan^{-1} x = \dfrac{1}{1+x^2}$.
(**Hint:** Rewrite $y = \tan^{-1} x$ as $\tan y = x$ and differentiate with respect to x using the chain rule.)

Improper Integrals (unbounded interval)

Let $f(x)$ be continuous on the interval $[a, \infty)$. If $\displaystyle\lim_{b\longrightarrow\infty}\int_a^b f(x)\,dx$ exists and has a finite value, we say that the improper integral $\displaystyle\int_a^\infty f(x)\,dx$ **converges** and define

$$\int_a^\infty f(x)\,dx := \lim_{b\longrightarrow\infty}\int_a^b f(x)\,dx$$

2 Determine if the improper integral $\displaystyle\int_{-1}^\infty \frac{1}{(x+6)^{3/2}}\,dx$ converges. If so, evaluate it.

3 Determine whether the integral $\displaystyle\int_e^\infty \frac{1}{x\ln x}\,dx$ is divergent or convergent. If it is convergent, evaluate it. If not, state your answer as "divergent."

Improper Integrals (unbounded integrand)

If f is continuous on $(a, b]$ and $\displaystyle\lim_{x\longrightarrow a^+} f(x) = \pm\infty$, we define

$$\int_a^b f(x)\,dx := \lim_{c\longrightarrow a^+}\int_c^b f(x)\,dx$$

provided that this limit exists.

4 Compute the value of the following improper integral $\displaystyle\int_1^e \frac{1}{x\sqrt{\ln x}}\,dx$.