MA 138 Worksheet #8

Section 7.4

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1 Determine whether the integral $\displaystyle\int_0^1 \frac{-3}{x^7}\,dx$ is divergent or convergent.

2 Determine whether the integral $\displaystyle\int_{-2}^2 \frac{2x}{(x^2-1)^{1/3}}\,dx$ is divergent or convergent. If it is convergent, evaluate it. If not, state your answer as "divergent."
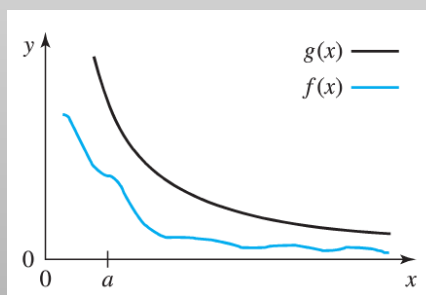

Convergence Test

We assume that $f(x) \geq 0$ for $x \geq a$.

To show that $\displaystyle\int_a^\infty f(x)\,dx$ is convergent

it is enough to find a function $g(x)$ such that

- $f(x) \leq g(x)$ for all $x \geq a$;

- $\displaystyle\int_a^\infty g(x)\,dx$ is convergent.

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3 (a) Show that $0 \leq e^{-x^2} \leq e^{-x}$ for $x \geq 1$.

   (b) Use your result in (a) to show that the integral $\displaystyle\int_1^\infty e^{-x^2}\,dx$ is convergent.

4 (a) Show that $0 \leq \dfrac{1}{\sqrt{1+x^4}} \leq \dfrac{1}{x^2}$ for $x > 0$.

   (b) Use your result in (a) to show that the integral $\displaystyle\int_1^\infty \frac{1}{\sqrt{1+x^4}}\,dx$ is convergent.

Divergence Test

We assume that $f(x) \geq 0$ for $x \geq a$.

To show that $\displaystyle\int_a^\infty f(x)\,dx$ is divergent

it is enough to find a function $g(x)$ such that

- $g(x) \leq f(x)$ for all $x \geq a$;

- $\displaystyle\int_a^\infty g(x)\,dx$ is divergent.