MA 138 Worksheet #9

Review for Exam 1 & Section 8.1

2/6/24

1 The exam has the same structure as previous exams. It covers the topics from Section 6.3 to Section 7.4 (Lectures 1 through 10). The bonus questions may be on the the topics of Section 8.1 (as covered in Lectures 11 and 12).

2 Make sure to be familiar with the types of problems discussed in the lectures, homework assignments, and recitation worksheets:

- Application of integration (Section 6.3):
  - average of a continuous function on an interval;
  - area between curves;
  - cumulative change.
- The substitution rule (Section 7.1)
- Integration by parts (Section 7.2)
- Rational functions and partial fractions (Section 7.3):
  - long division algorithm for polynomials;
  - case of a proper fraction with distinct linear factors in the denominator;
  - case of a proper fraction with repeated linear factors in the denominator;
  - case of a proper fraction with (possibly repeated) irreducible quadratic factors in the denominator;
  - the derivative of $y = \tan^{-1}(x)$.
- Improper integrals (Section 7.4):
  - unbounded interval of integration;
  - unbounded integrand over a finite interval of integration;
  - test for convergence and test for divergence for improper integrals.

3 Use the old exams as a guide to possible questions. The previous quizzes can also serve as a guide. Check the solutions provided online to see where you made mistakes in the previous quizzes.

Separable Differential Equations (Section 8.1)

We restrict ourselves to first-order differential equations of the form $\boxed{\frac{dy}{dx} = f(x)g(y).}$

That is, the right-hand side of the equation is the product of two functions, one depending only on x, $f(x)$, the other only on y, $g(y)$.

In order to solve the separable differential equation we divide both sides of the equation by $g(y)$ [assuming that $g(y) \neq 0$]:

$$\frac{1}{g(y)}\frac{dy}{dx} = f(x).$$

Now, if $y = u(x)$ is a solution of the equation, then $u(x)$ satisfies

$$\frac{1}{g[u(x)]}u'(x) = f(x).$$

If we integrate both sides with respect to x, we find that

$$\int \frac{1}{g[u(x)]}u'(x)\,dx = \int f(x)\,dx \qquad \text{or} \qquad \boxed{\int \frac{1}{g(y)}\,dy = \int f(x)\,dx}$$

since $g[u(x)] = g(y)$ and $u'(x)dx = dy$.

4 Find the solution of the differential equation $\dfrac{dy}{dx} + 0.3xy = 3x$ that satisfies the initial condition $y(0) = 5$.

5 Find the solution of the differential equation $\dfrac{dy}{dx} = \dfrac{\ln x}{xy}$ that satisfies the initial condition $y(1) = 5$.