MA 138
Calculus II with Life Science Applications
SECOND MIDTERM

Spring 2023
03/07/2023

Name: ______________

Sect. #: ______________

Do not remove this answer page — you will return the whole exam. No books or notes may be used. Use the backs of the question papers for scratch paper. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or a QWERTY keyboard is permitted. Absolutely no cell phone use during the exam is allowed.

The **first part of the exam** consists of 10 multiple choice questions, each worth 5 points. Record your answers on this page by filling in the box corresponding to the correct answer. For example, if (a) is correct, you must write

[a] b c d e

Do not circle answers on this page, but please do circle the letter of each correct response in the body of the exam. It is your responsibility to make it CLEAR which response has been chosen. You will not get credit unless the correct answer has been marked on **both** this page **and** in the body of the exam.

The **second part of the exam** consists of five open-response questions and one bonus question. When answering these questions, check your answers when possible. Clearly indicate your answer and the reasoning used to arrive at that answer. *Unsupported answers may receive NO credit.*

1. a b c d e

2. a b c d e

3. a b c d e

4. a b c d e

5. a b c d e

6. a b c d e

7. a b c d e

8. a b c d e

9. a b c d e

10. a b c d e

GOOD LUCK!

QUESTION	SCORE	OUT OF
Multiple Choice		50 pts
11.		10 pts
12.		10 pts
13.		10 pts
14.		10 pts
15.		10 pts
Bonus.		10 pts
TOTAL		100 pts

Please make sure to list the correct section number on the front page of your exam. In case you forgot your section number, consult the following table:

Sections #	Lecturer	Time/Location
001–004	Alberto Corso	MWF 10:00 am - 10:50 am, CB 110
Section #	**Recitation Instructor**	**Time/Location**
001	Kathryn Hechtel	TR 08:00 am - 08:50 am, CB 307
002	Kathryn Hechtel	TR 09:00 am - 09:50 am, CB 307
003	Davis Deaton	TR 10:00 am - 10:50 am, CB 307
004	Davis Deaton	TR 11:00 am - 11:50 am, CB 307

1. Which of the following differential equations is separable?

Possibilities:

(a) $\dfrac{dy}{dx} = \dfrac{2x + xy^2}{x + x^2 y}$

(b) $\dfrac{dy}{dx} = \dfrac{x + y}{2y}$

(c) $\dfrac{dy}{dx} = \dfrac{2y}{x^2 + 1}$

(d) $\dfrac{dy}{dx} = \dfrac{y^2}{2x - 3y}$

(e) None of the above

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2. The solution of the initial value problem

$$\frac{dy}{dx} = 3x^2y^2 \qquad \text{with} \qquad y(1) = 1$$

is

Possibilities:

(a) $y = \dfrac{1}{2 - x}$

(b) $y = \dfrac{1}{2 - x^2}$

(c) $y = \dfrac{1}{2 - x^3}$

(d) $y = \dfrac{1}{2 - x^4}$

(e) $y = \dfrac{1}{2 - x^5}$

3. The equation $y^2 = cx$ is the general solution of

(a) $\dfrac{dy}{dx} = \dfrac{2y}{x}$

(b) $\dfrac{dy}{dx} = \dfrac{y}{2x}$

(c) $\dfrac{dy}{dx} = \dfrac{2x}{y}$

(d) $\dfrac{dy}{dx} = \dfrac{x}{2y}$

(e) None of the above.

4. Two different strains of bacteria sometimes feed on chemicals excreted by one another, i.e. strain A feeds on chemical produced by strain B and vice versa. This phenomenon is referred to as *cross-feeding*. Suppose that the amount of strain A bacteria is governed by

$$\frac{dP}{dt} = P(1-P)(3P-6).$$

Using the analytic approach we see that the stable equilibrium point(s) is (are):

Possibilities:

(a) $\widehat{P}_1 = 0$ and $\widehat{P}_2 = 2$

(b) $\widehat{P} = 1$

(c) $\widehat{P} = 0$

(d) $\widehat{P} = 3$

(e) $\widehat{P}_1 = 0$ and $\widehat{P}_2 = 6$

5. Consider the following differential equations labeled **A.** through **D.**

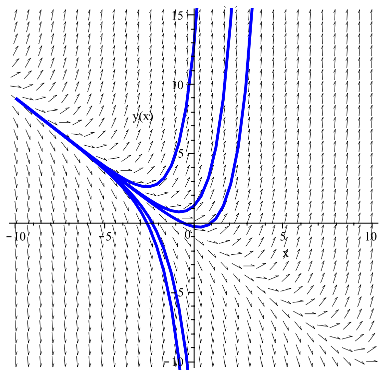
A. $y' = 2 - y$

B. $y' = x + y$

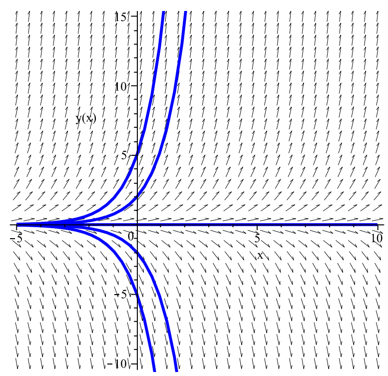
C. $y' = y$

D. $y' = \dfrac{10}{x}$

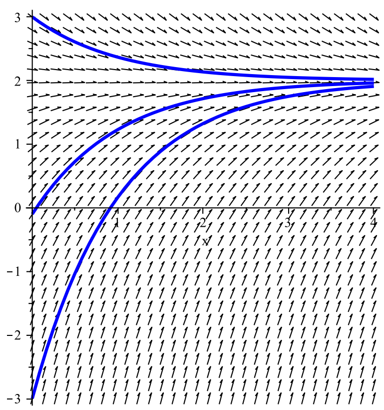
and consider the following slope (or direction) fields labeled I. through IV.



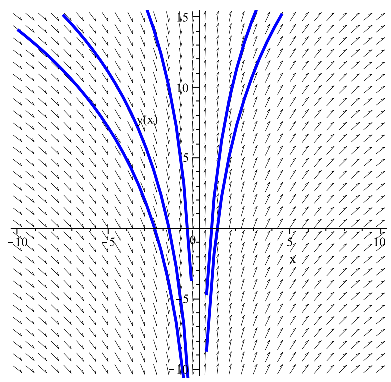
slope field I.



slope field II.



slope field III.



slope field IV.

Which slope field corresponds to differential equation **D.**?

(a) slope field I.

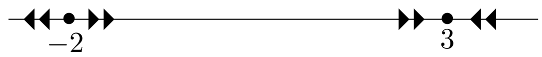
(b) slope field II.

(c) slope field III.

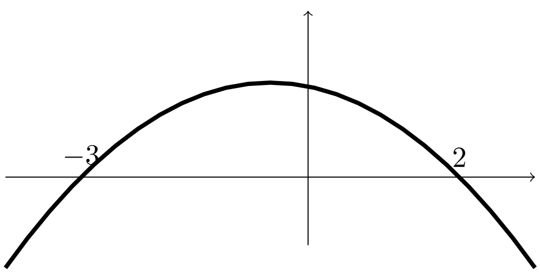
(d) slope field IV.

(e) None of the above

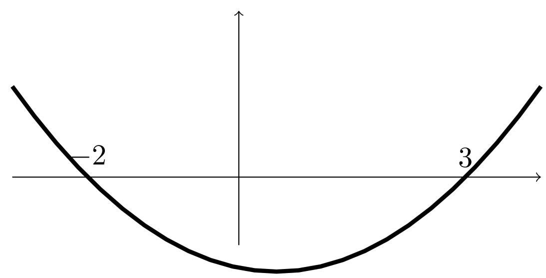
6. A phase line for an autonomous differential equation $\dfrac{dy}{dx} = f(y)$ is shown below



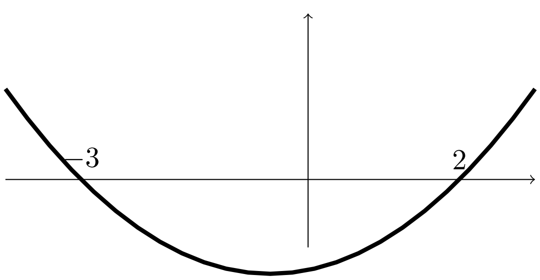
Which of the following graphs most closely matches the graph corresponding to the differential equation? **Note:** y and $f(y)$ are plotted on the horizontal and vertical axes, respectively



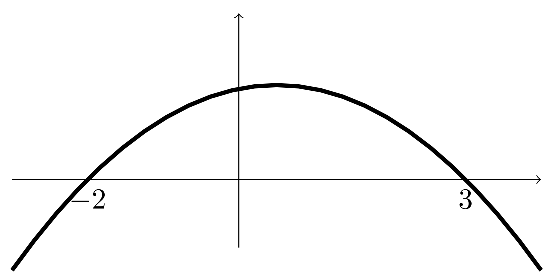
Graph A



Graph B



Graph C



Graph D

Possibilities:

(a) Graph A

(b) Graph B

(c) Graph C

(d) Graph D

(e) None of the above

7. The augmented matrix

$$\left[\begin{array}{ccc|c} 1 & -2 & 0 & 1 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 1 & 7 \end{array}\right]$$

can be put into the row-reduced form

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -7 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 7 \end{array}\right]$$

using just two row operations. Which row operations will accomplish this?
(**Note:** R_i denotes the entries on the i-th row)

Possibilities:

(a) First replace R_2 with $R_2 - R_3$, then replace R_1 with $R_1 - 2R_2$

(b) First replace R_1 with $R_1 + 2R_2$, then replace R_2 with $R_2 - R_3$

(c) First replace R_3 with $\frac{1}{7}R_3$, then replace R_2 with $\frac{1}{3}R_2$

(d) First replace R_2 with $R_3 - R_2$, then replace R_1 with $R_2 + 2R_1$

(e) First replace R_2 with $R_2 - R_3$, then replace R_1 with $R_1 + 2R_2$

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8. Suppose the augmented matrix of a linear system is row-equivalent to

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & 4 \\ 0 & 1 & -3 & 7 \\ 0 & 0 & k & h \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Which of the following must be true? (**Note:** More than one answer may be correct.)

Possibilities:

(a) If $k = 0$ and $h = -2$, the linear system has no solution.

(b) If $k = -2$ and $h = 0$, the linear system has no solution.

(c) If $k = 0$ and $h = 0$, the linear system has a unique solution.

(d) If $k = 0$ and $h = 0$, the linear system has infinitely many solutions.

(e) If $k = -2$ and $h = -2$, the linear system has infinitely many solutions.

9. Consider the matrix

$$A = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 0 & -1 \end{bmatrix}.$$

Then the matrix AA^T is:

(Recall that the notation A^T denotes the transpose of the matrix A. Our textbook uses the alternative notation A'.)

Possibilities:

(a) $AA^T = \begin{bmatrix} 6 & -1 \\ -1 & 0 \end{bmatrix}$

(b) $AA^T = \begin{bmatrix} 14 & -1 \\ -1 & 2 \end{bmatrix}$

(c) $AA^T = \begin{bmatrix} 5 & 2 & 5 \\ 2 & 1 & 3 \\ 5 & 3 & 10 \end{bmatrix}$

(d) $AA^T = \begin{bmatrix} 3 & 3 & 5 \\ 3 & 0 & 3 \\ 5 & 3 & 8 \end{bmatrix}$

(e) None of the above

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10. Consider the matrix $A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}.$

What is the fourth power of the matrix A, namely $A^4 = A \cdot A \cdot A \cdot A$?

Possibilities:

(a) $A^4 = \begin{bmatrix} 1^4 & 1^4 \\ 1^4 & 0 \end{bmatrix}$

(b) $A^4 = \begin{bmatrix} 4 & 4 \\ 4 & 0 \end{bmatrix}$

(c) $A^4 = \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}$

(d) $A^4 = \begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}$

(e) $A^4 = \begin{bmatrix} 8 & 5 \\ 5 & 3 \end{bmatrix}$

11. Separate variables and use the partial fraction method to solve the first order differential equation

$$\frac{dy}{dt} = \frac{1}{4}y(y-4)$$

with the following initial condition: $y = 2$ for $t = 0$.

(**Hint:** start from $\dfrac{4}{y(y-4)}dy = dt$ ...)

$y(t) =$ ____________________

Find $\lim_{t\to\infty} y(t) =$ ______

pts: /10

12. Consider the differential equation

$$\frac{dy}{dx} = \frac{1}{4}y(y-4)$$

that we already studied in Problem **11.**

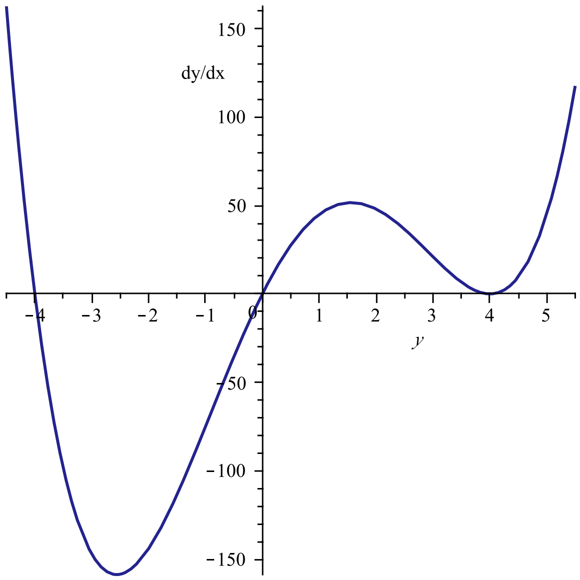
Find all equilibria $\widehat{y}$ of the above differential equation and discuss the stability of these equilibria using the Stability Criterion ($\equiv$ analytic approach).

pts: /10

13. Consider the differential equation

$$\frac{dy}{dx} = f(y)$$

where the graph of $f(y)$ is given below



Find all equilibria $\widehat{y}$ of the above differential equation and discuss the stability of these equilibria using the graphical approach.

pts: /10

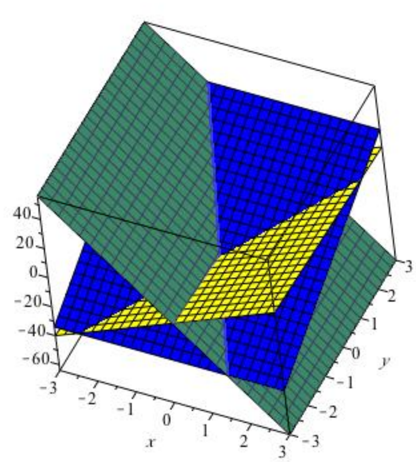
14. Solve the following system of linear equations

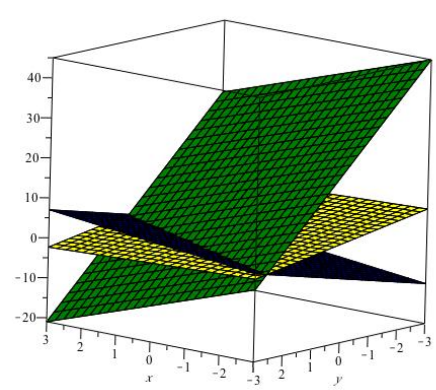
$$\left\{ \begin{array}{rcrcrcr} x & + & 4y & + & 3z & = & 8 \\ x & + & 2y & - & z & = & 2 \\ 3x & + & 8y & + & z & = & 12 \end{array} \right. ,$$

by writing the corresponding augmented matrix and then by row reducing.

How many solutions does the system have?

Which of the two pictures below illustrates the geometric situation described by the given system of linear equations?





pts: /10

15. Find the values of a and b that satisfy the following matrix equation:

$$\begin{bmatrix} 1 & 2a \\ 1 & 2 \end{bmatrix} \cdot \left(\begin{bmatrix} 0 & 2 \\ 6-a & 1 \end{bmatrix}^T \right) = \frac{1}{2} \begin{bmatrix} -40 & 2 \\ 8 & b \end{bmatrix}.$$

(Recall that the notation A^T denotes the transpose of the matrix A. Our textbook uses the alternative notation A'.)

pts: /10

Bonus. Consider the system

$$\begin{cases} 8x + 3y = 13 \\ 2x + y = 4. \end{cases}$$

(a) Write this system in matrix form $A \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$, where A is a 2×2 matrix and b_1, b_2 are appropriate numbers.

(b) Solve this system using the inverse of A.

(c) Check your answer.

pts: /10