STA 291
Lecture 8

- Probability
  - Probability Rules
  - Joint and Marginal Probability

Union and Intersection

- Let *A* and *B* denote two events.
- **The union of two events:**

$$A \cup B$$

- **The intersection of two events:**

$$A \cap B$$

Complement

- Let *A* denote an event.
- **The complement of an event *A*:**

A^C

Law of Complements:

$$P(A) = 1 - P(A^c)$$

Additive Law of Probability

Let A and B be two events in a sample space S. The probability of the union of A and B is

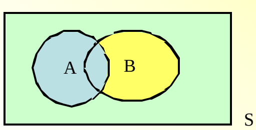
$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

Let A and B be two events in a sample space S. The probability of the union of two **disjoint** (**mutually exclusive**) events A and B is

$$P(A \cup B) = P(A) + P(B).$$

Using Additive Law of Probability

Example: At State U, all first-year students must take chemistry and math. Suppose 15% fail chemistry, 12% fail math, and 5% fail both. Suppose a first-year student is selected at random. What is the probability that student selected failed at least one of the courses? What is the probability that student pass both?



Disjoint Events

- Let *A* and *B* denote two events.
- **Disjoint (mutually exclusive) events:**

$$A \cap B = \varnothing$$

- No overlap

Probability tables

- Simple table: One row of outcomes, one row of corresponding probabilities.
- R x C probability tables: when the outcomes are classified by two features

- Gender and support President Obama?
- Smoker? And Lung disease?
- Age group and support Obama?

Example: Smoking and Lung Disease

	Lung Disease	No Lung Disease	*Marginal (smoke status)*
Smoker	0.12	0.19	0.31
Nonsmoker	0.03	0.66	0.69
Marginal (disease status)	0.15	0.85	1.0

Frequency table and probability table

	Lung Disease	No Lung Disease	*(total) Marginal (smoke status)*
Smoker	120	190	
Nonsmoker	30	660	
(total) Marginal (disease status)			1000

- Equivalent to a table with 4 entries:

(smoker & lung disease)	0.12
(smoker & not lung disease)	0.19
(nonsmoker & lung disease)	0.03
(nonsmoker & not lung disease)	0.66

But the R x C table reads much better

- From the R x C table we can get a table for smoker status alone, or disease status alone.
- Those are called marginal probabilities

It's a one way street

- Given the joint probability table, we can figure out the marginal probability
- Given the marginal, we may not determine the joint: there can be several different joint tables that lead to identical marginal.

Example: Smoking and Lung Disease

	Lung Disease	Not Lung Disease	*Marginal (smoke status)*
Smoker	0.02	0.29	0.31
Nonsmoker	0.13	0.56	0.69
Marginal (disease status)	0.15	0.85	

Same marginal, different joint.

Using the table

- P(smoker and lung disease) = 0.02
- P(smoker or lung disease) = 0.44

(either by looking at the table
Or using the additive rule for probability)

Independence of events

- May not always hold.
- If and when it hold: With independence, one way street becomes two way street.
- Smoking and lung disease are obviously not independent in reality.

Independence

- If events *A* and *B* are independent, then the events *A* and *B* have no influence on each other.
- So, the probability of *A* is unaffected by whether *B* has occurred.

Multiplication rule of probability

If A and B are two independent events, then

$$P(A \cap B) = P(A)P(B).$$

- i.e. joint prob. = product of two marginal prob.

Conditional Probability

$$P(A|B)=\frac{P(A\cap B)}{P(B)}\text{ , provided } P(B)\neq 0$$

- Note: $P(A|B)$ is read as "the probability that A occurs given that B has occurred."

Independent Events

Multiplication Rule for Independent Events: Let A and B be two independent events, then

$$P(A\cap B)=P(A)P(B).$$

Mathematically, if A is independent of B, then: $P(A|B) = P(A)$

Examples:

- Flip a coin twice. What is the probability of observing two heads?
- Flip a coin twice. What is the probability of getting a head and then a tail? A tail and then a head?

- In general, if events A and B are not independent, then the multiplication rule becomes

$$P(A\cap B)=P(A)\,P(B|A)$$

Terminology

- *P(AnB)=P(A and B)*
  Joint probability of *A* and *B*
  *(*of the intersection of *A* and *B)*
- *P(A|B)* Conditional probability of *A* given *B*
- *P(A)* (Marginal) probability of *A*

- If we have the probability table, then everything can be figured out from the table. NO need to use the rules.
- Only when no table is available, then we may be able to find out some probabilities from some given/known probabilities (a partial table) using rules.

- In homework/exam, you may be given a probability table, and are asked to verify certain rules.

Or

- Given a partial table, you are asked to use various rules to find the missing probabilities in the table.

Examples

Homework

Attendance Survey Question

- On a 4”x6” index card
  - Please write down your name and section number
  - Today’s Question:
  - Is A independent of B in reality?

A={Stock market go up today};
B={snow > 3 inch in New York today}