Homework 1

STA321 Spring 2009

Due Jan. 29

1. Let Y be a binomial (n, p) random variable with parameter p (n is known). Let $\hat{p} = Y/n$ Show that $(\hat{p})^2$ is NOT an unbiased estimator of p^2.

2. Let $X_1, \ldots, X_n$ be iid $\sim Bernoulli(p)$. Define $\hat{p} = (1/n)\sum X_i$.

   a) Compute the MSE for estimating p with $\hat{p}$. Show $\hat{p}$ is a consistent estimator of p. (we did this in class, the MSE is $p(1-p)/n$)

   b) Let $\tilde{p} = (0.5 + \sum X_i)/(n+1)$. Compute the MSE of $\tilde{p}$ and show $\tilde{p}$ is also consistent.

   c) When $n = 100$ and $p = 0.5$, which estimator ($\hat{p}$ or $\tilde{p}$) is the better estimator in terms of MSE? When $p = 0.1$ and $n = 100$, which is the better estimator?

   d) When $n = 100$, and $p = 0.01$, which estimator is the better estimator in terms of MSE?

3. Let $X_1, \cdots, X_n$ be iid $\sim N(\mu, \sigma^2)$ where both μ and σ^2 are unknown, $-\infty < \mu < \infty$, $0 < \sigma^2 < \infty$. (Please note, here the variance σ^2 is the parameter not the standard deviation σ)

   a) compute the MSE of estimator s^2 as an estimator of σ^2. (s^2 is the so called sample variance).

   b) Let us try a new estimator of σ^2: $\hat{\sigma}^2 = Cs^2$, for some positive constant C.

   Find the best constant C in the sense that minimizes the MSE.