Solution to Prob 2:

$$\log Lik = \log \prod_{i=1}^{n} \theta^2 y_i e^{-\theta y_i} = \sum_i \left(2\log\theta + \log Y_i - \theta Y_i\right)$$

$$\frac{\partial}{\partial\theta} \log Lik = 2n \cdot \frac{1}{\theta} + 0 - \Sigma y_i = \frac{2n}{\theta} - \Sigma y_i$$

set $\frac{\partial}{\partial\theta}\log Lik = 0$, we get $\hat{\theta}_{mLE} = \frac{2n}{\Sigma Y_i}$ this is mLE

Now the test statistic

$$-2 \log \frac{Lik(\theta=1)}{Lik(\theta=\hat{\theta}_{mLE})} = -2\left[\left(\cancel{\Sigma \log Y_i} - \Sigma Y_i\right) - \left(2n \log \hat{\theta}_{mLE} + \cancel{\Sigma \log Y_i} - \hat{\theta}_{mLE}\Sigma Y_i\right)\right]$$

$$= -2\left[\left(\hat{\theta}_{mLE} - 1\right)\Sigma Y_i - 2n \log \hat{\theta}_{mLE}\right]$$

from R

$$\left(\hat{\theta}_{mLE} = 50/\Sigma Y_i = 0.7918409\right)$$

$$= -2\left[(0.7918409 - 1)\, 63.144 - 50 \cdot \log 0.7918409\right]$$

$$= 2.948517.$$

This corresponds to
P-value = 0.086 [in R > 1-pchisq(2.9485, df=1)]

So, not enough evidence to reject $H_0: \theta = 1$.