Homework 1, Sta 531 Fall 2008

Due 9/2

1. Find the probability of at least one 6 in 4 rolls of a fair dice. Find the probability of at least a pair of 6 in 24 rolls of two fair dices.
2. Problem 1.2 (b), (d)
3. Problem 1.9 (b)
4. Given a sequence of events, A_i, show that

$$\bigcap_{k=3}^{\infty} \bigcup_{i>k} A_i = \{\omega | \omega \in A_i \text{ for infinite many index } i \ \}$$

5. For a given sample space S and one proper, non-empty subset A. Claim: the smallest σ algebra that include A is given by

$$\mathcal{B} = \{\emptyset, A, A^c, S\}.$$

   Prove this is an σ algebra.

6. For a given sample space S and two proper, non-empty subsets A and B. Assume $A \neq B, A^c \neq B$. Explicitly give the smallest $\sigma-$algebra that include A and B. (list all the subsets.)

Hidden in the proof of the Theorem 1.2.11 (p. 12) is the following Proposition:
Given any sequence of increasing measurable sets $B_1, B_2, \cdots$, we have

$$P(\bigcup_{i=1}^{\infty} B_i) = \lim_{n\to\infty} P(B_n) = \lim_{n\to\infty} P(\bigcup_{i=1}^{n} B_i) \ .$$

If we denote $\bigcup_{i=1}^{\infty} B_i$ by $\lim\limits_{n\to\infty} B_n$ then the above is just

$$P(\lim_{n\to\infty} B_n) = \lim_{n\to\infty} P(B_n).$$