#1 (problem 14 on page 277)

$\mathbb{P}_1$: only one class. $\{0,1,2\}$ and recurrent.

$\mathbb{P}_2$: only one class $\{0,1,2,3\}$ and recurrent.

$\mathbb{P}_3$: $\{0,2\}$ recurrent, $\{3,4\}$ recurrent, $\{1\}$ transient.

$\mathbb{P}_4$: $\{0,1\}$ recurrent, $\{2\}$ recurrent, $\{3\}$ transient, $\{4\}$ transient.

#2. model this as a MC with two states: good, bad.

$$\mathbb{P} = \begin{array}{c} \\ \text{good} \\ \text{bad} \end{array} \begin{array}{c} \begin{array}{cc} \text{good} & \text{bad} \end{array} \\ \begin{pmatrix} 1/2 & 1/2 \\ 1/3 & 2/3 \end{pmatrix} \end{array}.$$

(a) In year 0, initial condition $(1,0)$ i.e. 100% a good year at year 0.

In year 1: $(1,0)\cdot\mathbb{P} = (\frac{1}{2}, \frac{1}{2})$ so, it is equally likely a good/bad.

In year 2: $(1,0)\mathbb{P}^2 = (\frac{5}{12}, \frac{7}{12})$.

So, the expected total # of storms in year 1 is: $\frac{1}{2}\cdot 1 + \frac{1}{2}\cdot 3 = 2$

the expected for year 2 is: $\frac{5}{12}\cdot 1 + \frac{7}{12}\cdot 3 = 13/6 = 2\frac{1}{6}$.

(b) In year 3, the distribution of good/bad is $(1,0)\mathbb{P}^3 = (\frac{58}{144}, \frac{86}{144})$.

and thus

So the chance of No storm in year 3 is:

$$\frac{58}{144} \times \frac{e^{-1}\, 1^0}{0!} + \frac{86}{144} \times \frac{e^{-3}\, 3^0}{0!} = \frac{58}{144} e^{-1} + \frac{86}{144} e^{-3} \approx 0.168$$

#3.

(a) $P(N(t)=n) = 0.3 \cdot e^{-3t} \frac{(3t)^n}{n!} + 0.7\, e^{-5t} \frac{(5t)^n}{n!} =$

(b) $N(t)$ is a mix of 2 Poisson processes. [not a Poisson process]

(c) $N(t)$ has stationary increments. Look at (5.27), it does not depend on s.
(OR same for all s.)

(d) $N(t)$ do not have indep. increment property. Reason was given on page 351 below eq (5.27)

(e) This is similar to a Bayesian Question.

$P(\text{good}) = 0.3$, $P(\text{bad}) = 0.7$; $P(3 \text{ storm by } t=1 \mid \text{good}) = e^{-3} \frac{3^3}{3!}$

$P(3 \text{ storm by } t=1 \mid \text{bad}) = e^{-5} \frac{5^3}{3!}$

We want to find

$$P(\text{good} \mid 3 \text{ storm by } t=1) = \text{Bayes formula} = 0.406$$

This also illustrate the "non-indep" nature of the process.
[since $0.406 \neq P(\text{good}) = 0.3$]