The video game analogy of two sample log-rank tests

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Consider two teams of m and n players (boys vs. girls).

1. All the players may begin a video game by putting down \$1 on the table. (the ticket price).

Suppose now all players enter and start play at the same time. When the first player failed among the $n+m$, his/her \$1 on the table will be divided equally among all the players (that are active playing, including himself/herself), and he/she will be disqualified from further competition and leave the room with \$ $1/(n+m)$.

In general, when the k^{th} player failed, his/her \$1 on the table will be divided equally among all the players **that are in the room playing at the time**, including himself/herself, and then he/she will be disqualified from further competition and leave the room with his/her earned money.

The total earnings of the girls team is of interest. In fact, **This is log-rank statistic.** (if the total net earnings of the girls team is ≈ 0 then there is no significant difference between the two teams.)

2. If girls and boys are equally good at the video game, the expected total earnings for the girls team (the \$1 she has to pay counts as negative earning) is **zero**. (hint: consider each girl's individual earnings)

3. At any time if a player wants to quit (=censoring) before been "killed in the game", it is **fair** to let him/her take the \$1 on the table back and keep all his/her current earnings.
[since E(future earning |current earnings, alive) = 1] (in fact it is a so called *martingale in time t*).

Where **fair** means, if he/she keeps playing, his/her expected **future** earnings is \$1, so let him/her grab the \$1 and quit is fair.

4. At any time if a player wants to join the game, all he/she needs to do is to put \$1 on the table and start playing (just like starting a new game). In fact a player can get in or out of the game multiple times and the game is still fair, (assume he/she cannot see into future). This is sometime called "late entry", or "switch treatment" or "time-change covariate".

(play for the girls team for a while, quit, and later re-join to play for the boys team = switch).

5. The game may stop when people are still actively playing. (= force those to censor; = study ends with those patients still alive.) People forced to quit get their \$1 back.

[**Proof that total earnings of girls team is log rank test**] We have known the log rank test from other context as

$$\int \frac{R_1 R_2}{R_1+R_2}\left[\frac{dN_1}{R_1} - \frac{dN_2}{R_2}\right] = \int \frac{R_2 dN_1}{R_1+R_2} - \int \frac{R_1 dN_2}{R_1+R_2} \ .$$

Let us focus on the first integration of the second expression. The integration is only non zero when $dN_1 > 0$; and if $dN_1(t_k) > 0$, then $R_2 \frac{dN_1}{R_1+R_2}$ represents the team #2's (or sample#2)'s earning at time t_k. (since the death is/are from sample #1, team #2 got positive earning).

Similarly, when there is a death from sample #2, the team #2's earning is $-\frac{R_i dN_2}{R_1+R_2} = R_2 \frac{dN_2}{R_1+R_2} - dN_2$, from the second integration. So, the integration represent the total earning over time for team #2.