Non parametric Bayes.

How to define a random density fn. $f(x)$?

This can be easy. But we want to have certain "good" properties.

For example: Given two densities $f_1(x)$ and $f_2(x)$, define

$$f(x) = p \cdot f_1(x) + (1-p) \cdot f_2(x)$$ where p is a Beta random variable.

This is a random density since p is random.

To make the so defined $f(x)$ richer [cover more ground], we can use k given densities $f_1(x), f_2(x) \cdots f_k(x)$ and a random vector $(p_1, p_2 \cdots p_k)$ that have Dirichlet distribution, and define

$$f(x) = \sum_{i=1}^{k} p_i f_i(x)$$ this is random because $(p_1 p_2 \cdots p_k)$ is random

To push one step futher, Let k to be random, with a distribution like Poisson [but minus $\{0\}$].

Finally, what $f_1(x)$ $f_2(x) \cdots$ sequence we should use ?

We want the sequence to have following property: that the linear Combination of $f_1(x) \cdots f_k(x)$ can approx. ANY densities.

[If $g(x)$ is any density, we can find $(a_1 a_2 \cdots a_k)$, s.t. $\left| g(x) - \sum_{i=1}^{k} a_i f_i(x) \right| < \varepsilon$

beta densities on the interval $[0,1]$ have this property.

$\forall K$, define

$$f_{K1}(x) = \text{beta}(1,k) \; ; \; f_{K2}(x) = \text{beta}(2,k) \; \cdots \; f_{KK}(x) = \text{beta}(K,1)$$

To summarize:

① Given (fix) a distribution on positive integes. [Like Poisson \o]

Generate a random variable K.

② Given K, generate a vector of probabilities

$(p_1\ p_2 \cdots p_K) \sim$ from Dirichlet distribution.

③ using the beta density sequence, define a random density

$$f(x) = \sum_{i=1}^{K} p_i f_{Ki}(x) \quad \cdots\cdots \quad (\ast)$$

This random density $f(x)$ has the following property.

For any density $g(x)$ on $(0,1)$, there is always a density of the form $(\ast)$ that for any $\varepsilon > 0$

$$\sup_{0<x<1} | g(x) - f(x) | < \varepsilon$$

this say the densities of $(\ast)$ type is everywhere [dense].