Homework 4 - Due 10:00 AM on Friday August 9

Solutions should be clear and organized. Make sure you justify your work.

1. Prove or disprove: If $\sum_{n=1}^{\infty} a_n$ is convergent and $\sum_{n=1}^{\infty} b_n$ is divergent, then $\sum_{n=1}^{\infty} (a_n + b_n)$ is divergent.

2. Find a value of c such that $\sum_{n=1}^{\infty} (1+c)^{-n} = 2$.

3. Determine whether the series is convergent or divergent.

   (a) $\sum_{n=1}^{\infty} \frac{n^2+2}{(n+1)^2}$

   (b) $\frac{1}{1} + \frac{1}{2\sqrt{2}} + \frac{1}{3\sqrt{3}} + \frac{1}{4\sqrt{4}} + \cdots$

   (c) $\sum_{n=6}^{\infty} \frac{\sqrt{n}}{n-5}$

   (d) $\sum_{n=2}^{\infty} \frac{1}{n(\ln(n))^2}$

   (e) $\sum_{n=1}^{\infty} \left(\int_{n}^{n+1} \frac{dx}{x^{5/3}} \right)$

   (f) $\sum_{n=1}^{\infty} \frac{n^2+1}{n^3+1}$

4. Can you find a sequence $\{a_n\}$ converging to 0 such that the series $\sum_{n=1}^{\infty} a_n$ diverges?

5. Find the Maclaurin series representation for each of the following series. (Hint: It is unnecessary to take any derivatives.)

   (a) $f(x) = \frac{1}{x+10}$

   (b) $f(x) = \frac{x}{2x^2+1}$

6. Evaluate the indefinite integral as a power series.

$$\int \frac{\ln 1 - t}{t}\, dt$$

7. Find the Taylor series for each of the following functions at the indicated center.

   (a) $\cos(x), \quad c = \pi/4$

   (b) $e^x + e^{-x}, \quad c = 0$