Homework 5 - Due 10:00 AM on Monday August 12
Solutions should be clear and organized. Make sure you justify your work.

1. Let V be the set of all linear combinations of the vectors $\begin{bmatrix} 2 \\ 5 \end{bmatrix}$ and $\begin{bmatrix} 7 \\ 1 \end{bmatrix}$. Is V a vector space? Prove or disprove.

2. Let W be the set of points on the line $y = 2x + 1$. That is, let
$$W = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} : y = 2x + 1 \right\}.$$
Is W a vector space? Prove or disprove.

3. Prove that the set of all differentiable functions on $\mathbb{R}$ is a vector space over $\mathbb{R}$.

4. Prove that the set of all rational numbers is NOT a vector space over $\mathbb{R}$.

5. Prove that the map $T : \mathbb{R}^2 \to \mathbb{R}^3$ that maps a vector $\mathbf{v}$ to the vector $\mathbf{0}$ is a linear transformation.

6. Suppose that T is the linear transformation $T : \mathbb{R}^4 \to \mathbb{R}^5$ that maps a vector
$$\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} \text{ to } \begin{bmatrix} -v_1 - 7v_3 \\ v_2 \\ v_1 \\ 0 \\ v_2 - v_3 \end{bmatrix}.$$
Find a matrix A such that $T(\mathbf{v}) = A\mathbf{v}$.

7. Suppose that T is the linear transformation $T : \mathbb{R}^5 \to \mathbb{R}^6$ that takes the coefficient vector $\mathbf{v} = \begin{bmatrix} v_0 \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix}$ of a polynomial $f(x) = v_0 + v_1 x + v_2 x^2 + v_3 x^3 + v_4 x^4$ and maps it to the coefficient vector of $\int_0^x f(x)\,dx$. Find a matrix A such that $T(\mathbf{v}) = A\mathbf{v}$.

8. Is the vector $\mathbf{w} = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$ in $\text{Span}\left\{ \begin{bmatrix} -4 \\ 7 \\ 8 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \right\}$?