Homework 6 - Linear Independence, Basis, and Eigenvalues Practice

Make sure to justify your solution for each problem.

1. Determine if the columns of A form a linearly independent set.

$$A = \begin{bmatrix} -4 & -3 & 0 \\ 0 & -1 & 4 \\ 1 & 0 & 3 \\ 5 & 4 & 6 \end{bmatrix}$$

2. Prove that if $S = \{\mathbf{v_1}, \mathbf{v_2}, \ldots, \mathbf{v_p}\}$ is a linearly dependent set of vectors in $\mathbb{R}^n$, then there exists $\mathbf{v_k}$ in S such that $\text{Span}(S \setminus \{\mathbf{v_k}\}) = \text{Span}(S)$.

3. Find a basis for each of the following subspaces of $\mathbb{R}^n$.

   (a) All vectors whose components are equal in $\mathbb{R}^4$.

   (b) All vectors whose components add up to zero in $\mathbb{R}^4$.

4. Consider the matrix $A = \begin{bmatrix} 2 & 5 & -8 & 7 \\ -1 & 5 & 4 & 7 \\ 0 & 5 & 0 & 7 \end{bmatrix}$.

   (a) Find two different bases for $ColA$.

   (b) Find two different bases for $NulA$.

5. Suppose S is a 5-dimensional subspace of $\mathbb{R}^6$. Prove that every basis for S can be extended to a basis for $\mathbb{R}^6$ by adding one more vector.

6. Find the eigenvalues of

$$B = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}.$$

7. Prove that the eigenvalues of A are the same as the eigenvalues of A^T for any square matrix A.