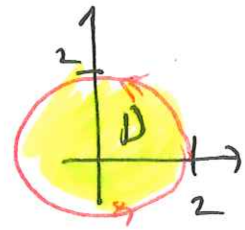
$$\oint_C \left(x^2y\,dx - xy^2\,dy\right)$$

$C: \; x^2+y^2=4$



$$\oint P\,dx + Q\,dy$$

$D: \; 0 \le r \le 2$
$\;\;\;\; 0 \le \theta \le 2\pi$

$$P(x,y) = x^2y$$
$$Q(x,y) = -xy^2$$

$$\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = -y^2 - x^2$$
$$= -(x^2+y^2)$$

By Green

$$\oint_C x^2y\,dx - xy^2\,dy = \iint_D -(x^2+y^2)\,dA$$
$$= \int_0^{2\pi}\int_0^2 (-r^2)\,r\,dr\,d\theta$$
$$= 2\pi\left[-\frac{r^4}{4}\right]\Bigg|_{r=0}^{r=2} = \frac{2\pi}{4}(-16)$$
$$= -8\pi$$

(4) $z=0$ $\quad \vec{F} = \langle 0, y, 0 \rangle$

$\vec{n} = \langle 0, 0, -1 \rangle \quad \vec{F} \cdot \vec{n} = 0$

So:

$$\iint_{(1)} \vec{F} \cdot d\vec{S} = \iiint_E \operatorname{div} \vec{F} \cdot dV - \iint_{(2)} \vec{F} \cdot dS$$

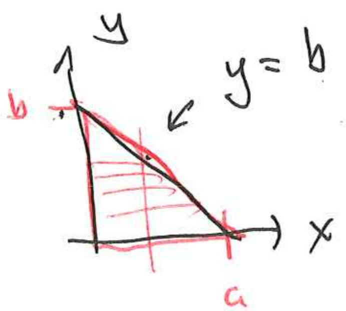
$$\vec{F} = \langle z, y, zx \rangle$$

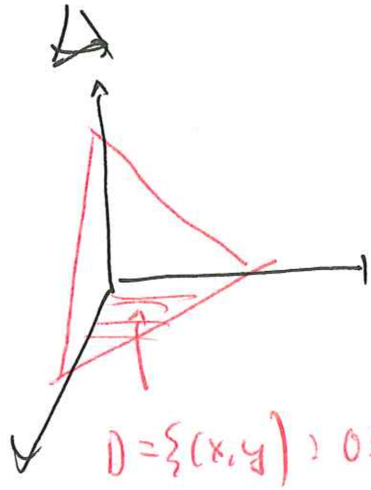
$$\operatorname{div} \vec{F} = \frac{\partial}{\partial x}(z) + \frac{\partial}{\partial y}(y) + \frac{\partial}{\partial z}(xz)$$

$$= 0 + 1 + x$$

$$\iiint_E \operatorname{div} F \cdot dV = \iiint_E (1+x)\, dV$$

~~Region where z=0:~~



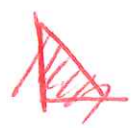


$D = \{(x,y) : 0 \le x \le a,$
$0 \le y \le b - \frac{b}{a}x\}$

$$\iiint_E \operatorname{div} F \, dV = \iiint_E (1+x)\, dV$$

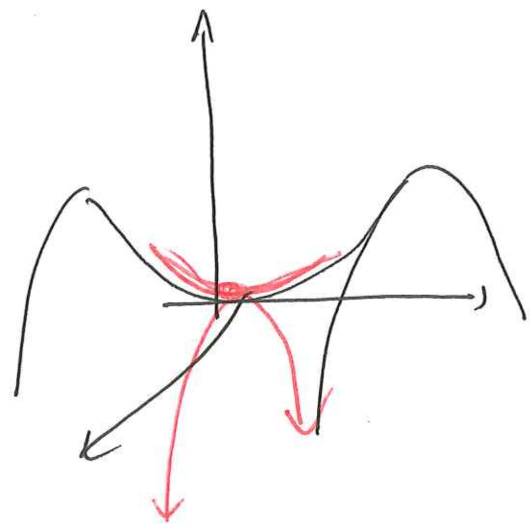
$$= \int_0^a \int_0^{b-\frac{b}{a}x} \int_0^{c(1-\frac{x}{a}-\frac{y}{b})} (1+x)\, dz\, dy\, dx$$

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$



$$\frac{z}{c} = 1 - \frac{x}{a} - \frac{y}{b}$$

$$\boxed{z = c\left(1 - \frac{x}{a} - \frac{y}{b}\right)}$$



---

① $f(x,y) = x^2 - y^2$

$\frac{\partial f}{\partial x} = 2x \qquad \frac{\partial f}{\partial y} = 2y \Rightarrow$ One CP $(0,0)$

$f_{xx} = 2 \qquad f_{xy} = 0 \qquad D = \begin{vmatrix} 2 & 0 \\ 0 & -2 \end{vmatrix} = -4$

$f_{yx} = 0 \qquad f_{yy} = -2$

saddle point

---

② on bdry $x = \cos t \quad y = \sin t$

$\phi(t) = f(\cos t, \sin t) = \cos^2 t - \sin^2 t$

$\phi'(t) = 2\cos t(-\sin t) - 2\sin t \cos t$

$= -4\cos t \sin t$

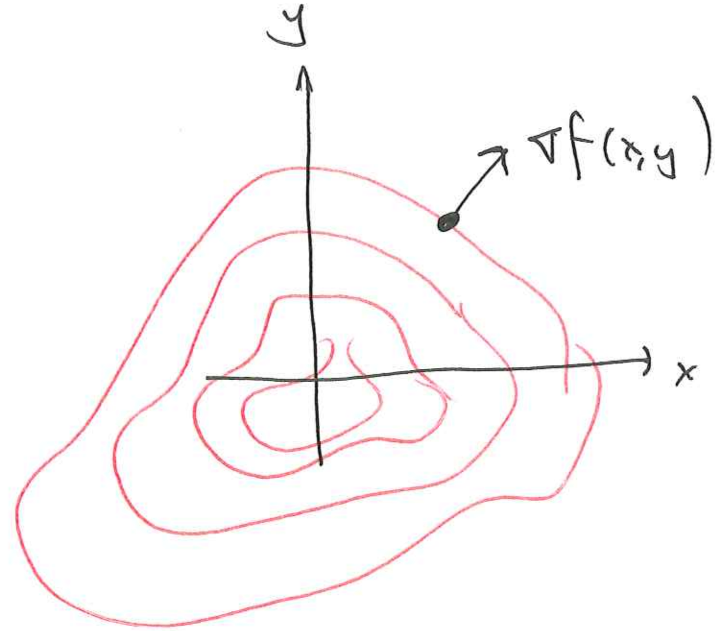
$\downarrow = 0 \quad t = 0, \frac{\pi}{2}, \pi,$

$3\pi/2, 2\pi$

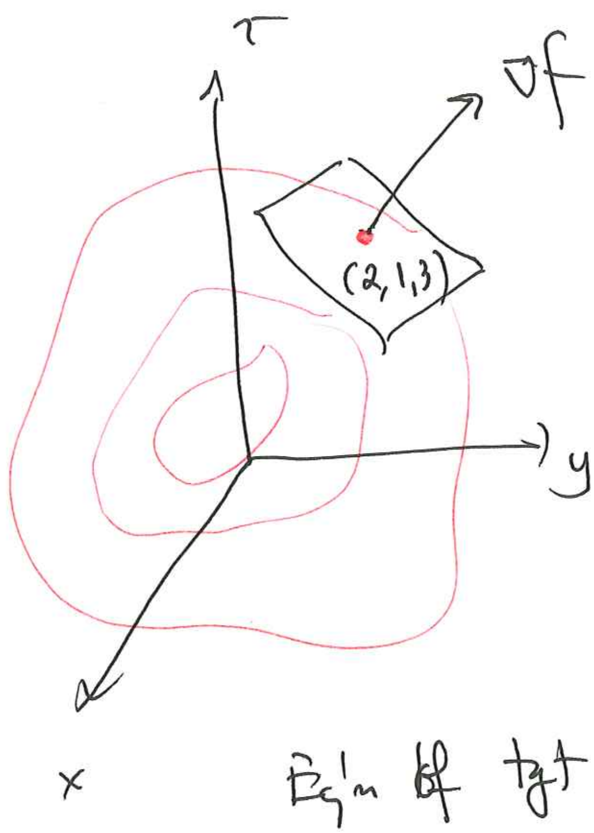
t	$\phi(t)$
0	1
$\pi/2$	-1
π	1
$3\pi/2$	-1
2π	1

Extreme vals: are ± 1

level curves



level surfaces



$f(x,y,z) = x^2+4y^2+z^2$

$\nabla f = \langle 2x, 8y, 2z \rangle$

$\nabla f(2,1,3) =$

$\langle 4, 8, 6 \rangle$

Eq'n of tgt plane:

$4(x-2) + 8(y-1) + 6(z-3) = 0$

$$x(u,v) = u^2 - v^2$$

$$y(u,v) = 2uv$$

$$\frac{\partial x}{\partial u} = 2u \qquad \frac{\partial x}{\partial v} = -2v$$

$$\frac{\partial y}{\partial u} = 2v \qquad \frac{\partial y}{\partial v} = 2u$$

$$J = \begin{vmatrix} 2u & -2v \\ 2v & 2u \end{vmatrix} = 4u^2 + 4v^2$$