MA 114 Worksheet #09: Sequences

1. (a) Give the precise definition of a **sequence**.
   (b) What does it mean to say that $\lim_{x\to a} f(x) = L$ when $a = \infty$? Does this differ from $\lim_{n\to\infty} f(n) = L$? Why or why not?
   (c) What does it means for a sequence to converge? Explain your idea, not just the definition in the book.
   (d) Sequences can diverge in different ways. Describe two distinct ways that a sequence can diverge.
   (e) Give two examples of sequences which converge to 0 and two examples of sequences which converges to a given number $L \neq 0$.

2. Write the first four terms of the sequences with the following general terms:
   (a) $\dfrac{n!}{2^n}$
   (b) $\dfrac{n}{n+1}$
   (c) $(-1)^{n+1}$
   (d) $\{a_n\}_{n=1}^{\infty}$ where $a_n = \dfrac{3}{n}$.

3. Find a formula for the nth term of each sequence.
   (a) $\left\{\dfrac{1}{1}, -\dfrac{1}{8}, \dfrac{1}{27}, -\dfrac{1}{64}, \ldots\right\}$
   (b) $\left\{1, \dfrac{1}{2}, \dfrac{1}{4}, \dfrac{1}{8}, \dfrac{1}{16}, \ldots\right\}$

4. Suppose that a sequence $\{a_n\}$ is bounded above and below. Does it converge? If not, find a counterexample.

5. The limit laws for sequences are the same as the limit laws for functions. Suppose you have sequences $\{a_n\}$, $\{b_n\}$ and $\{c_n\}$ with $\lim_{n\to\infty} a_n = 15$, $\lim_{n\to\infty} b_n = 0$ and $\lim_{n\to\infty} c_n = 1$. Use the limit laws of sequences to answer the following questions.
   (a) Does the sequence $\left\{\dfrac{a_n \cdot c_n}{b_n + 1}\right\}_{n=1}^{\infty}$ converge? If so, what is its limit?
   (b) Does the sequence $\left\{\dfrac{a_n + 3 \cdot c_n}{2 \cdot b_n + 2}\right\}_{n=1}^{\infty}$ converge? If so, what is its limit?

6. Write out the first five terms of
   (a) $a_0 = 0$, $a_1 = 1$ and $a_{n+1} = 3a_{n-1} + {a_n}^2$.
   (b) $a_1 = 6$, $a_{n+1} = \dfrac{a_n}{n}$.
   (c) $a_1 = 2$, $a_{n+1} = \dfrac{a_n}{a_n + 1}$.
   (d) $a_1 = 2$, $a_2 = 1$, and $a_{n+1} = a_n - a_{n-1}$.