Calculus II   Russell Brown
Exam 1   10 February 2004

Answer all of the following questions. Additional sheets are available if
necessary. No books or notes may be used. You may use a calculator. You
may not use a calculator which has symbolic manipulation capabilities.
When answering these questions, please be sure to 1) check answers when
possible, 2) clearly indicate your answer and the reasoning used to arrive at
that answer *(unsupported answers may not receive credit)*.
Each question is followed by space to write your answer. Please lay out
your solutions neatly in the space below the question. You are not expected
to write each solution next to the statement of the question.

Name _____

Section _____

Question	Score	Total
1		6
2		8
3		15
4		10
5		15
6		18
7		10
8		6
9		12
Total		100

1. If $\int_4^9 f(x)\,dx = 5$, find the value of $\int_2^3 xf(x^2)\,dx$.

2. Let $f(x) = \csc(x)$, the cosecant function, for $0 < x \leq \pi/2$.

   (a) Sketch the graph of $f(x)$ and give the domain and range of the function f.

   (b) On the same axes, sketch the graph of the inverse function $f^{-1}(x)$ and give the domain and range of the function f^{-1}.
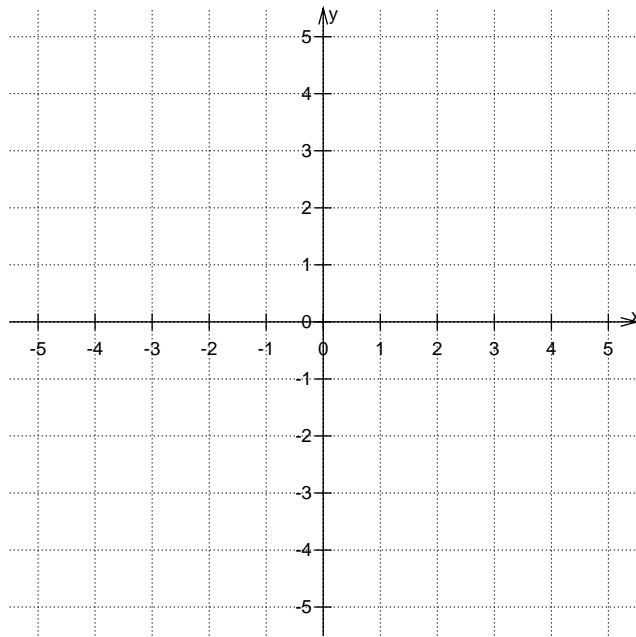
3. Let $f(x) = xe^{x^2}$ for x a real number and let g be the inverse function to f.

  (a) Use calculus to show that the function $f(x)$ is one-to-one.
  (b) Find $f(1)$.
  (c) Find $g(e)$.
  (d) Find $g'(e)$.

4. (a) Compute the first four derivatives of $f(x) = xe^x$.

   (b) Guess a formula for the nth derivative of $f(x)$, $f^{(n)}(x)$. (Of course, you are eager to prove this by induction. But please don't–you may need the time for the later problems.)

5. A sample of Kryptonite 210 has mass 300 grams at time $t = 0$ and decays to 100 grams after 180 days.

   (a) Find the mass of the sample for all time.

   (b) Find the mass after 100 days.

   (c) Find the half-life of Kryptonite 210.

6. Use calculus to evaluate the following integrals.

(a) $\int_0^{1/2} \frac{1}{\sqrt{1 - x^2}} \, dx$

(b) $\int \frac{x + 1}{x^2 + 4} \, dx$

(c) $\int \frac{\cos x}{\sin x} \, dx$

7. (a) Simplify the expression $\sin^{-1}(\cos(7\pi/6))$.

   (b) If $\cos u = 5/13$ and $-\pi \le u \le 0$, find $\sin u$, $\tan u$ and $\cos 2u$.

8. Give a complete statement of L'Hopital's rule for the indeterminate form 0/0.

9. Find the following limits.

(a) $\lim_{x\to 0} \frac{e^x}{x}$

(b) $\lim_{x\to 0} \frac{\ln{(1+2x)}}{x}$

(c) $\lim_{x\to 0} \frac{e^x - 1 - x}{x^2}$

(d) $\lim_{x\to 0}(1 + 3x)^{1/x}$