Calculus II — Russell Brown

Exam 2 — 21 October 2003

Answer all of the following questions. Additional sheets are available if necessary. No books or notes may be used. You may use a calculator. You may not use a calculator which has symbolic manipulation capabilities.
When answering these questions, please be sure to 1) check answers when possible, 2) clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may not receive credit).*
Each question is followed by space to write your answer. Please lay out your solutions neatly in the space below the question. You are not expected to write each solution next to the statement of the question.

Name ______________________

Section ____________

Question	Score	Total
1		35
2		15
3		15
4		10
5		10
6		10
7		12
Min(Total,100)		107

1. Evaluate the following integrals.

   (a) $\displaystyle\int_0^{\pi/2} x\sin(2x)\,dx$

   (b) $\displaystyle\int \sin^3(x)\cos^3(x)\,dx$

   (c) $\displaystyle\int_0^{1/2} \frac{1}{\sqrt{1-x^2}}\,dx$

   (d) $\displaystyle\int_0^{2} \frac{1}{x^2}\,dx$

   (e) $\displaystyle\int \frac{x}{\sqrt{4-x^2}}\,dx$

   (f) $\displaystyle\int \frac{x}{x^2+4x+5}\,dx$

   (g) $\displaystyle\int \frac{1}{2+\sqrt{x+1}}\,dx$

2. for each of the following functions, write out the form of the partial fractions decomposition. Do not solve for the coefficients.

   (a) $\dfrac{x}{(x+1)(x+4)}$

   (b) $\dfrac{x^2+1}{x^4+x^3+2x^2}$

   (c) $\dfrac{x}{x^4+2x^2+1}$

3. (a) Find the partial fractions decomposition of the function

$$\frac{1}{x^4 + x^2}.$$

(b) Find the anti-derivative

$$\int \frac{1}{x^4 + x^2}\, dx.$$

4. The trapezoid rule T_n and Simpson's rule S_n for approximating the integral $\int_a^b f(x)\,dx$ are

$$\begin{aligned} T_n &= \frac{h}{2}(f(x_0) + 2f(x_1) + \ldots + 2f(x_{n-1}) + f(x_n)) \\ S_n &= \frac{h}{3}(f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \ldots + 4f(x_{n-1}) + f(x_n)) \end{aligned}$$

The error in the trapezoid rule, E_T and in Simpson's rule, E_S, satisfy

$$|E_T| \leq \frac{K_2(b-a)^3}{12n^2} \quad \text{and} \quad |E_S| \leq \frac{K_4(b-a)^5}{180n^4}$$

where K_j is a number so that the jth derivative satisfies $|f^{(j)}(x)| \leq K_j$ for all x with $a \leq x \leq b$.

(a) Use the trapezoid rule with with $n = 5$ to estimate the integral

$$\int_0^2 e^{-x^2}\,dx.$$

Round your answer to 3 decimal places.

(b) Use Simpson's rule with $n = 4$ to estimate the integral

$$\int_0^2 e^{-x^2}\,dx.$$

Round your answer to 3 decimal places.

5. (a) Find a value of n so that the error in approximating the integral

$$\int_1^3 e^{-3x}\,dx$$

by the trapezoid rule T_n is less than 10^{-4}.

(b) Find a value of n so that the error in approximating the integral

$$\int_1^3 e^{-3x}\,dx$$

by Simpson's rule S_n is less than 10^{-4}.

6. Find the area of the region

$$\{(x, y) : x \leq 1, \text{ and } 0 \leq y \leq e^x\}.$$

7. (a) State the comparison theorem for improper integrals.

   (b) Use the comparison theorem to determine if the following integrals converge or diverge.

      i. $\displaystyle\int_0^\infty \frac{\sin^2(x)}{1+x^2}.$

      ii. $\displaystyle\int_1^\infty \frac{2+e^{-x}}{1+x}\,dx.$