Calculus II | Russell Brown
Exam 2 | 7 March 1900

Answer all of the following questions. Use the answer sheets provided. Additional sheets are available if necessary. No books or notes may be used. You may use a calculator. When answering these questions, please be sure to 1) check answers when possible, 2) clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may receive NO credit)*. 3) There are 105 points on this exam, however if you earn more than 100 points, you will only be given a score of 100.

Name ______________________

Section ____________

Question	Score	Total
1		50
2		10
3		10
4		10
5		10
6		15
min(Total,100)		100

1. Compute the following integrals, if possible. If an improper integral diverges, say so.

   (a) $\int_0^{\pi/2} \sin^3 x \, dx$

   (b) $\int \sin^2 x \, dx$

   (c) $\int_0^{\pi/2} \frac{\sin x}{1+\cos^2 x} \, dx$

   (d) $\int_0^1 xe^x \, dx$

   (e) $\int \frac{x^2}{x^2+4} \, dx$

   (f) $\int \frac{1}{x^2+x^3} \, dx$

   (g) $\int \frac{1}{x^2+x} \, dx$

   (h) $\int_0^1 \frac{1}{\sqrt{1-x^2}} \, dx$

   (i) $\int_0^{\pi/2} \tan x \, dx$

   (j) $\int_0^{\infty} \frac{1}{1+x^2} \, dx$

2. Which of the integrals a), c), h), i) and j) are improper? For each of these integrals, give the point or points where it is improper.

3. (a) Give the definition of a rational function. Give an example of a rational function.

   (b) Give the definition of a proper rational function. Give examples of a proper rational function and an improper rational function.

4. (a) Find the anti-derivative

$$\int \sqrt{1-x^2}\,dx.$$

(b) Use your answer to compute

$$\int_0^1 \sqrt{1-x^2}\,dx,$$

(c) The integral in part b) represents the area of a familiar region. Use a geometric argument to give the area and check your answer to part b).

5. (a) State L'Hopital's rule for the indeterminate form 0/0. Give both the hypotheses and the conclusion.

   (b) Give an example of two functions f and g where

$$\lim_{x\to 1} \frac{f(x)}{g(x)} \neq \lim_{x\to 1} \frac{f'(x)}{g'(x)}.$$

6. Compute the following limits:

   (a) $\lim_{x\to 0^+} x \ln x$

   (b) $\lim_{x\to 0} (1+2x)^{1/x}$

   (c) $\lim_{x\to\infty} \frac{x^2}{e^x}$