Calculus II | Russell Brown
Exam 2 | 5 March 2002

Answer all of the following questions. Additional sheets are available if necessary. No books or notes may be used. You may use a calculator. You may not use a calculator which has symbolic manipulation capabilities.
When answering these questions, please be sure to 1) check answers when possible, 2) clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may not receive credit).*
Each question is followed by space to write your answer. Please lay out your solutions neatly in the space below the question. You are not expected to write each solution next to the statement of the question.
The total on this test is 105. However, no student will earn more than 100.

Name ______________________________

Section _______________

Question	Score	Total
1		40
2		15
3		15
4		15
5		15
6		5
Min(Total,100)		100

1. Compute 5 of the 6 indefinite integrals below. Write here ______ the letter of the integral that is not to be graded. If you do not specify the an integral which is not to be graded, we will take the five lowest scores.

   (a) $\int \sin^2 x \, dx$

   (b) $\int \sin^3 x \, dx$

   (c) $\int \frac{1}{(4 - x^2)^{3/2}} \, dx$

   (d) $\int \frac{x}{x^2 + 2x + 2} \, dx$

   (e) $\int x \cos(2x) \, dx$

   (f) $\int \frac{1}{1 + \sqrt{x}} \, dx$

2.

3. Find the values of λ for which $y(x) = e^{\lambda x}$ satisfies the equation

$$y'' - 4y' - 5y = 0.$$

4. Find the form of the partial fraction decomposition for the following rational functions. DO NOT SOLVE FOR THE CONSTANTS.

   (a) $\frac{x}{(x^2 - 4)(x^2 + 4)}$

   (b) $\frac{x^2 - 43x}{(x + 1)^3 (x + 2)(x^2 - 4)}$

   (c) $\frac{x + 1}{x^2 (x^2 + 1)^2 (x^2 + 2x + 1)}$

5. The trapezoid rule T_n and Simpson's rule S_n for approximating the integral $\int_a^b f(x)\,dx$ are

$$\begin{aligned} T_n &= \frac{h}{2}(f(x_0) + 2f(x_1) + \ldots + 2f(x_{n-1}) + f(x_n)) \\ S_n &= \frac{h}{3}(f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \ldots + 4f(x_{n-1}) + f(x_n)) \end{aligned}$$

The errors satisfy

$$|E_T| \le \frac{M_2(b-a)^3}{12n^2} \quad \text{and} \quad |E_S| \le \frac{M_4(b-a)^5}{180n^4}$$

where M_k is a number which satisfies $|f^{(k)}(x)| \le M_k$ for all x with $a \le x \le b$.

(a) Use the trapezoid rule and Simpson's rule with $n = 4$ to approximate the integral

$$\int_3^6 \sin(2x)\,dx.$$

Give your answers correctly rounded to four decimal places.

(b) Find n so that the error in the trapezoid rule is at most 10^{-4},

$$\left| \int_3^6 \sin(2x)\,dx - T_n \right| \le 10^{-4}.$$

(c) Find n so that the error in Simpson's rule is at most 10^{-4},

$$\left| \int_3^6 \sin(2x)\,dx - S_n \right| \le 10^{-4}.$$

6. (a) State the comparison theorem for improper integrals.

   (b) Use the comparison theorem to determine if the following improper integrals converge.

      i. $\displaystyle\int_1^\infty \sin^2(x)e^{-x}\,dx$

      ii. $\displaystyle\int_1^\infty \frac{2+\sin x}{x}\,dx$

7. (a) Find a function $y = y(x)$ which satisfies

$$\frac{dy}{dx} = y(2-y) \quad \text{and} \quad y(0) = 1.$$

(b) Find

$$\lim_{x \to \infty} y(x).$$

8. There are thirty-two teams in the 2002 FIFA Copa del Mundo. Thus, it is interesting to know the value of the integral

$$\int_0^\infty x^{32}e^{-x}\,dx.$$

Suppose that we know that

$$\int_0^\infty x^{31}e^{-x}\,dx = A.$$

Find a simple expression involving A which gives the value of the integral

$$\int_0^\infty x^{32}e^{-x}\,dx.$$

It is known that
$A = 8,222,838,654,177,922,817,725,562,880,000,000$, but I doubt this is very helpful.