Calculus II   Russell Brown
Exam 2   9 March 2004

Answer all of the following questions. Additional sheets are available if
necessary. No books or notes may be used. You may use a calculator. You
may not use a calculator which has symbolic manipulation capabilities.
When answering these questions, please be sure to 1) check answers when
possible, 2) clearly indicate your answer and the reasoning used to arrive at
that answer *(unsupported answers may not receive credit)*.
Each question is followed by space to write your answer. Please lay out
your solutions neatly in the space below the question. You are not expected
to write each solution next to the statement of the question.

Name _____

Section _____

Question	Score	Total
1		32
2		10
3		10
4		10
5		10
6		10
7		10
8		12
min(Total,100)		100

1. Use calculus to evaluate the following integrals.

   (a) $\int_0^\pi \cos^2 x \, dx$

   (b) $\int x \sin(2x) \, dx$

   (c) $\int \frac{\sin x}{1 + \cos^2 x} \, dx$

   (d) $\int \frac{1}{(9 - x^2)^{3/2}} \, dx$

2. (a) Define *rational function.*

   (b) Give an example of a rational function.

   (c) Define *irreducible quadratic polynomial.*

   (d) Give an example of an irreducible quadratic polynomial.

3. Give the form of the partial fractions decomposition for the following functions. DO NOT SOLVE FOR THE CONSTANTS.

(a) $\dfrac{1}{(x^2 + 1)(x - 2)^2}$

(b) $\dfrac{x}{(x^2 - 2x + 1)^2}$

4. Determine if each of the integrals is convergent or divergent and evaluate the convergent improper integrals.

(a) $\displaystyle\int_0^\infty \frac{1}{1+x^2}\,dx$

(b) $\displaystyle\int_1^\infty \frac{dx}{x}$

5. (a) State the comparison theorem.

   (b) Use the comparison theorem to determine if the integral
   $$\int_0^\infty \frac{1 + \sin^2 x}{2 + x} \, dx$$
   converges or diverges.

6. The trapezoid rule T_n for approximating the integral $\int_a^b f(x)\,dx$ is

$$T_n \;=\; \frac{h}{2}\left(f(x_0) + 2f(x_1) + \ldots + 2f(x_{n-1}) + f(x_n)\right)$$

The error in the trapezoid rule, E_T, satisfies

$$|E_T| \leq \frac{K_2(b-a)^3}{12n^2}$$

where K_2 is a number so that the 2nd derivative satisfies
$|f^{(2)}(x)| \leq K_2$ for all x with $a \leq x \leq b$.

(a) Use the trapezoid rule with $n = 5$ to approximate the integral

$$\int_0^1 \frac{1}{1+x^2}\,dx$$

(b) Compute the exact value of the integral to help check your answer.

7. Find n so that the difference between the trapezoid rule T_n and the exact value of the integral

$$\int_0^{10} e^{-2x} \, dx$$

is at most 10^{-3}.

8. (a) Solve the initial value problem

$$y' = y(3 - y) \quad y(0) = 1.$$

(b) Find

$$\lim_{t \to \infty} y(t).$$