Calculus II — Russell Brown
Exam 3 — 18 November 2003

Answer all of the following questions. Additional sheets are available if necessary. No books or notes may be used. You may use a calculator. You may not use a calculator which has symbolic manipulation capabilities. When answering these questions, please be sure to 1) check answers when possible, 2) clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may not receive credit).*
Each question is followed by space to write your answer. Please lay out your solutions neatly in the space below the question. You are not expected to write each solution next to the statement of the question.

Name _____

Section _____

Question	Score	Total
1		12
2		8
3		18
4		10
5		10
6		5
7		20
8		10
9		12
min(score,100)		100

1. Suppose that a tank holds 500 liters of water initially. Brine flows in at a rate of 5 liters/minute. The brine has a concentration of 4 grams/liter. The tank is well-stirred and brine flows out a rate of 5 liters/minute.

   (a) Write a differential equation for $M(t)$, the mass of salt in the tank after t minutes.

   (b) Solve the differential equation you wrote in part a) and find the function $M(t)$ for all $t \geq 0$.

   (c) Find the mass of salt in the tank after 15 minutes.

   (d) Find $\lim_{t \to \infty} M(t)$.

2. Find the length of the graph of $f(x) = x^{3/2}$ for $2 < x < 3$.

3. Determine if the following statements are true or false and write true or false in the spaces provided.

   (a) If $\lim_{n\to\infty} a_n = 0$, then the series $\sum_{n=1}^{\infty} a_n$ is convergent.

   (b) If $\lim_{n\to\infty} a_n = 34$, then the series $\sum_{n=1}^{\infty} a_n$ is divergent.

   (c) The series $\sum_{n=1}^{\infty} 2^n$ is convergent.

   (d) If $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = 1$, then the series $\sum_{n=1}^{\infty} a_n$ converges.

   (e) $1 = \sum_{n=1}^{\infty} \frac{9}{10^n}$.

   (f) For every series in the universe, we have

$$|\sum_{n=1}^{\infty} a_n - \sum_{n=1}^{N} a_n| \leq |a_{N+1}|.$$

   a. _____ b. _____ c. _____ d. _____ e. _____ f. _____

4. Determine if the following sequences converge and find the limit of each convergent sequence.

   (a) $a_n = \frac{n^2+\sin(n)}{2n^2-n}$.

   (b) $b_n = \cos(\pi n)$.

   (c) $c_n = \frac{2^n}{n!}$.

5. Determine if each series is convergent. If the series is convergent, give the value of the series.

   (a) $\sum_{n=1}^{\infty} \frac{1}{4n^2-1}$

   (b) $\sum_{n=3}^{\infty} 3^{-n}$

6. Give the definition of a conditionally convergent series.

7. Determine if each series is absolutely convergent, conditionally convergent or divergent. Briefly justify each answer.

   (a) $\sum_{n=1}^{\infty} \frac{n}{n^2+2}$.

   (b) $\sum_{n=1}^{\infty} \frac{\sin(n)}{n^2}$.

   (c) $\sum_{n=1}^{\infty} \frac{(-1)^n}{2n+1}$.

   (d) $\sum_{n=1}^{\infty} \frac{n^2}{3^n}$

8. Use the integral test to find a value N so that

$$\sum_{n=N}^{\infty} \frac{1}{n^3} \leq 10^{-4}.$$

9. Determine the radius and interval of convergence for the following power series. Briefly justify each answer.

   (a) $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n!}$

   (b) $\sum_{n=1}^{\infty} \frac{x^n}{n^2}$