Calculus II
Exam 3

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Answer all of the following questions. Use the backs of the question papers for scratch paper. Additional sheets are available if necessary. No books or notes may be used. When answering these questions, please be sure to 1) check answers when possible, 2) clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may receive NO credit)*.

Name ____________________

Section __________

Question	Score	Total
1		10
2		10
3		20
4		15
5		10
6		20
7		15
Total		100

1. Find the limits of the following sequences:

   (a) $\lim_{n\to\infty} \dfrac{n^2-1}{n^2+1}$.

   (b) $\lim_{n\to\infty} ne^{-n}$.

2. (a) Find a simple expression for the sum

$$S_N = 3 + 1 + \frac{1}{3} + \frac{1}{9} + \ldots + \frac{1}{3^N} = \sum_{k=-1}^{N} 3^{-k}.$$

(b) Use your answer in part a) to find

$$\lim_{N\to\infty} \sum_{k=-1}^{N} 3^{-k}.$$

3. Determine if each of the following series converges absolutely, converges conditionally or diverges. Indicate which test or tests you use.

   (a) $\displaystyle\sum_{k=1}^{\infty} k3^{-k}$

   (b) $\displaystyle\sum_{k=1}^{\infty} 3^{k}$

   (c) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$

   (d) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^n}{n^2}$

4. For each of the power series, give the radius of convergence and the interval of convergence. Indicate which test you use. Be sure to check the endpoints.

   (a) $\displaystyle\sum_{k=0}^{\infty} \frac{x^{2n}}{n!}$

   (b) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^n x^n}{n}$

5. Use the integral test to find a number m so that

$$\sum_{k=10}^{\infty} \frac{1}{k^4} \leq m.$$

Draw a picture to explain why your calculations give an upper bound. Be sure to label your axes!

6. Give the terms through x^5 for the MacLaurin series for the following functions:

   (a) $\sin x$

   (b) $\dfrac{\sin x}{x}$

   (c) e^{-x^2}

   (d) $\displaystyle\int_0^x e^{-t^2}\,dt$

   (e) $\sqrt{1+x}$

7. (a) State Taylor's theorem with remainder.

   (b) Give the MacLaurin polynomial $P_4(x)$ for $\cos x$ with terms through x^4.

   (c) Use the polynomial in part b) to find an approximation to $\cos(1/4)$.

   (d) Use the remainder from Taylor's theorem to find a bound for the error, $\cos(1/4) - P_4(1/4)$. That is, to find a number m with

$$|\cos\frac{1}{4} - P_4(\frac{1}{4})| \leq m.$$