Calculus II — Russell Brown

Exam 3 — 9 April 2002

Answer all of the following questions. Additional sheets are available if necessary. No books or notes may be used. You may use a calculator. You may not use a calculator which has symbolic manipulation capabilities. When answering these questions, please be sure to 1) check answers when possible, 2) clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may not receive credit)*.
Each question is followed by space to write your answer. Please lay out your solutions neatly in the space below the question. You are not expected to write each solution next to the statement of the question.

Name __________________________

Section _____________

Question	Score	Total
1		10
2		10
3		15
4		10
5		15
6		15
7		15
8		15
min(Total,100)		100

1. Find the following limits.

   (a) $\lim_{n \to \infty} \dfrac{2n^2 - 1}{3n^2 + 987}$

   (b) $\lim_{n \to \infty} \dfrac{n!}{(n+1)!}$

2. Find the sum of each of the following series.

   (a) $\dfrac{1}{3 \cdot 2} + \dfrac{1}{3 \cdot 4} + \dfrac{1}{3 \cdot 8} + \dfrac{1}{3 \cdot 16} + \ldots + \dfrac{1}{3 \cdot 2^n} + \ldots$

   (b) $\sum_{n=1}^{\infty} \left(\dfrac{1}{n} - \dfrac{1}{n+1} \right)$

3. Determine if each of the following series is absolutely convergent, conditionally convergent or divergent. Describe briefly how you test the series for convergence or divergence.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{3n+1}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(2n)!}$

   (c) $\displaystyle\sum_{n=1}^{\infty} (-3)^n$

4. Let

$$s = \sum_{n=1}^{\infty} \frac{1}{n^5} \quad \text{and} \quad s_N = \sum_{n=1}^{N} \frac{1}{n^5}.$$

Use the integral test to find a value of N so that $|s - s_N| < 10^{-5}$.

5. Find the radius and interval of convergence for the following power series.

   (a) $\displaystyle\sum_{n=0}^{\infty} \frac{(x-2)^n}{3^n}$.

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{(2n+1)}$

6. Find a power series representation for each of the following functions. Your answer should be a series with terms $c_n x^n$.

   (a) $\dfrac{1}{1-x}$

   (b) $\dfrac{1}{1-x^2}$

   (c) $\dfrac{x}{1-x^2}$

7. (a) Write the MacLaurin series or Taylor series about 0 for e^x.

   (b) Use your answer in part a) to find the MacLaurin series for e^{-x^2}.

   (c) Find the MacLaurin series for

$$\int_0^x e^{-t^2}\,dt.$$

8. (a) State one of the following carefully: Taylor's formula with remainder, or the alternating series estimation theorem.

(b) Find a value of N so that

$$|\frac{1}{e} - \sum_{n=0}^{N} \frac{(-1)^n}{n!}| \leq 10^{-3}.$$

Hint: You may use either the alternating series test, or Taylor's theorem with remainder.