Calculus II   Russell Brown

Exam 3   13 April 2004

Answer all of the following questions. Additional sheets are available if
necessary. No books or notes may be used. You may use a calculator. You
may not use a calculator which has symbolic manipulation capabilities.
When answering these questions, please be sure to 1) check answers when
possible, 2) clearly indicate your answer and the reasoning used to arrive at
that answer *(unsupported answers may not receive credit)*.
Each question is followed by space to write your answer. Please lay out
your solutions neatly in the space below the question. You are not expected
to write each solution next to the statement of the question.

Name ⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯

Section ⎯⎯⎯⎯⎯⎯⎯⎯

Question	Score	Total
1		10
2		6
3		10
4		6
5		6
6		20
7		8
8		20
9		20
min(Total,100)		100

1. Find the length of the curve $y = x^{3/2}$ for $1 \le x \le 4$.

2. For each of the following sequences, state whether or not the sequence is bounded and whether or not the sequence is monotone.

   (a) $1, 2, 3, 4, 5, \ldots$

   (b) $1, 1/2, 1/3, 1/4, \ldots$

   (c) $1, -1, 1, -1, 1, -1, \ldots$

3. Find the limits of the following sequences.

   (a) $\displaystyle \lim_{n \to \infty} n! e^{-n}$

   (b) $\displaystyle \lim_{n \to \infty} \frac{n^5}{n^8 + 1}$

4. Write the repeating decimal $0.\overline{41}$ as a fraction.

5. Find the sum of the series $\displaystyle\sum_{n=2}^{\infty} 4 \cdot 5^{-n}$.

6. Determine if the following sequences are conditionally convergent, absolutely convergent or divergent. Briefly explain your reasoning.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{4^n}{n!}$.

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{1+n^2}$.

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$

7. Compare with an integral to find an number M so that

$$\sum_{n=43}^{\infty} \frac{1}{n(\ln n)^2} \le M.$$

Draw a picture to justify your answer.

8. Give the radius and interval of convergence for each of the following series. Briefly explain your reasoning.

(a) $\sum_{n=1}^{\infty} \frac{3^n x^n}{n^2}$

(b) $\sum_{n=1}^{\infty} \frac{(x+1)^n}{n!}$

9. (a) Write the function $g(x) = \int_0^x \frac{1}{1+t^2}\, dt$ as a power series.

   (b) Give the radius of convergence for this series.

   (c) State the alternating series estimation theorem.

   (d) Use the alternating series to test to find an approximation to $g(1/4)$ which is correct to within an error of at most $1/1000$.

   (e) Use calculus to find the exact value of the integral. (You might want to ask if your answers to d) and e) are consistent.)